INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

Investigating the Effects of Integration on the Development of Mathematical Knowledge for Teaching: A Case of Pre-Service Teachers in the Colleges of Education in Ghana amidst Curriculum Changes

Bliss Cornelius Sedega
Vice Principal, Department of Mathematics, Science and ICT,
Mount Mary College of Education, Ghana

Abstract:
The study examined the effects of integration on the development of Mathematical Knowledge for Teaching (MKT) among pre-service teachers while enrolled in a math program at the colleges of education in Ghana in the face of the new curriculum. In all, two hundred and forty-seven (247) pre-service teachers (PSTs) were sampled for the study. The experimental group comprised one hundred and thirty (130) PSTs who were taught both Mathematics content and method courses in an integrated manner, while the control group, comprised of one hundred and seventeen (117) PSTs, was taught Mathematics content and method courses separately. The non-equivalent static-group comparison design was used for the study. Both groups of PSTs were post-tested after they had received instruction in mathematics courses up to the point where they were ready to go for their macro teaching (Off Campus Teaching Practice) to compare their achievements in terms of their mathematical knowledge for teaching. Analysis of data using the independent sample t-test revealed that the control group had a mean score lower than the experimental group, and the standard deviation showed that the scores of the control group were far from their mean score. That is, the scores of the control group were more heterogeneous than that of the experimental group. The experimental group scored higher than the control group on the post-test, and their scores were more homogeneous than the control group. At a significant level of 0.05, there was a statistically significant difference between the means of the two groups on the post-test. With a mean difference of 5.042, the experimental group performed better than the control group on the post-test. Hence, the students in the experimental group turned out to achieve better on MKT components than the students in the control group. It was recommended, among others, that the use of the integrated approach in the teaching and learning of Mathematics should be encouraged, monitored, and supported throughout the Colleges of Education in Ghana.

Keywords: *Mathematical knowledge for teaching, integration, mathematics content/method courses, content knowledge, pedagogical content knowledge, conceptual understanding*

1. Introduction

Teacher educators throughout the United States and Canada have been investigating the influence of teacher training programs on pre-service teachers' Mathematical Knowledge for Teaching (MKT) (Cardetti & Truxaw, 2014; Goodson-Epsy et al., 2014; Kajander & Holm, 2016; Tyminski, Zambak, Drake, & Land, 2014). MKT details a teacher's understanding of mathematics, content knowledge (CK), and the teaching and learning of mathematics or pedagogical content knowledge (PCK) (Ball, Thames, & Phelps, 2008). Specific to teaching practices, Ball et al. (2008) identified domains of MKT, such as specialized content knowledge and knowledge of content and students, that describe a teacher's ability to select appropriate tasks, anticipate errors, and design instruction to advance learning. Recent curriculum changes at the Colleges of Education in Ghana have prescribed that both content and methods be integrated during teaching and learning. The old curriculum, however, prescribed pre-service teachers' exposure to math content and pedagogy coursework in their own rights. That is, the teaching and learning of content and methods as separate courses. Initial observation at the study sites by the researcher during the 2019/2020 and 2020/2021 academic years showed that the concept of integrating content and methodology during teaching and learning as prescribed by the new curriculum poses a new challenge, especially to the tutors. Most mathematics educators at the study sites were seen focusing on teaching the content with varied teaching strategies without the usual rigid methodology whereby artefacts are arranged for the learners to interact with or manipulate to come out with some mathematical concepts. A few of the tutors were also seen focusing on teaching the methodology for the greater part of the lessons, even though all these tutors were supposed to be teaching the same courses but in different classrooms. Some of the tutors were seen using certain new strategies such as songs, games, plays, storytelling, and simulation in the teaching and learning process, which most of the learners found challenging trying to connect the strategies to the Mathematical concepts to be developed. It was also observed that the tutors themselves, with difficulty, tried to demonstrate the connection between these new strategies and the concepts to be developed. A failure to develop MKT during teacher preparation may negatively influence teacher candidates' future instructional practices and students' achievement. The depth of understanding of content and instructional practices of a teacher impacts the potential for students' learning. CK alone is insufficient to support the teaching of mathematics, and a lack of PCK negatively affects a teacher's instructional practice (Baki & Arslan, 2016; Bartell, Webel, Bowen, & Dyson, 2013; Maher & Muir, 2013). Teachers' MKT is linked to student achievement (Baki & Arslan, 2016; Hill, Umland, Litke, & Kapitula, 2012; Leong, Meng, & Abdul Rahim, 2015). Developing the CK and PCK of prospective teachers is essential to promote the successful teaching and learning of mathematics. Therefore, it is critical that teacher education programs ensure that curriculum requirements intentionally address MKT development. Teacher preparation programs have used a mixture of content and methods of coursework. A century ago, a central focus of teacher education in the United States was on developing a thorough understanding of the subject matter. However, the mid-twentieth century witnessed a steady shift to an emphasis on pedagogy generalized to be largely independent of subject matter. By the 1980s, an absence of content focus was so prevalent that Shulman (1986) referred to this as a 'missing paradigm' in teacher education. A similar tendency can be seen in other countries. For example, a few decades ago, it was possible to become qualified for teaching mathematics in grades 1–9 in Norway with no more mathematics than a short course in didactics. A widespread assumption seemed to be that prospective teachers already knew the content they needed from their experiences as students, and they only required directions on how to teach this content. Shulman's call for increased attention to subject matter reoriented research and practice. However, the connection between the formal education of mathematics teachers and the content understanding important for their work is not straightforward. Teachers' formal mathematics education is not highly correlated with their students' achievement or with the depth of understanding; they seem to have of the mathematical issues that arise in teaching. One of Shulman's (1986) most important contributions was the suggestion that the work of teaching requires professional knowledge that is distinctive for the teaching profession. He proposed different categories of professional knowledge for teaching. One of these categories was distinctive content knowledge, which Shulman described as including a deep knowledge of the structures of the subject beyond procedural and factual knowledge. Another category of knowledge was what Shulman termed "pedagogical content knowledge," which is aspects of the content most germane to its teaching (1986, p. 9). The idea of an amalgam of subject matter knowledge and pedagogical knowledge has continued to appeal to researchers working in different subject areas, and Shulman's foundational publications are among the most cited references in the field of education (Google Scholar identifies over 13000 publications that cite his 1986 article.) In the last two decades, researchers and mathematics educators have increasingly emphasized the significance of mathematical knowledge that is teaching-specific. Such knowledge is seen as different from the mathematics typically taught in most collegiate mathematics courses and from the mathematics needed by professionals other than teachers. Although it includes knowing the mathematics taught to students, the kind of understanding of the material needed by teachers is different from that needed by the students. Even though the literature suggests a general consensus that mathematics teaching requires special kinds of mathematical knowledge, an agreement is lacking about definitions, language, and basic concepts. Many scholars draw on Shulman's notion of pedagogical content knowledge (PCK) and view this knowledge as being either a kind of 'combined' knowledge or a kind of 'transformed' knowledge. Grounded in Shulman's proposals, the phrases 'for teaching' and 'practice-based' have been emphasized to indicate the relationship of the knowledge to specific work of teaching (e.g., Ball, Thames, & Phelps, 2008). With growing interest in ideas about specialized professional content knowledge, the early 2000s saw a spate of large-scale efforts to develop measures of such knowledge and the use of such a construct as the basis for a wide range of research studies, such as evaluating professional development, examining the impact of structural differences on the mathematical education of teachers (e.g., Kleickmann et al., 2015), arguing for policies and programs (e.g., Hill & Charalambous, 2012), and investigating the role of professional content knowledge on mathematics teaching practice (e.g., Smith et al. 2012).

This special issue about measuring PSTs development of mathematical knowledge for teaching in the colleges of education in Ghana, especially when both content and methods are presented in an integrated format, continues to be the broader purpose of this study and potential ways to advance the field. In an effort to situate this special issue, the study focused on what is being studied, how, and to what ends. Presenting content and instructional practices in a blended form affects the development of MKT more than addressing the concepts individually (Auslander, Smith, Hart, & Carothers, 2016; Hoover, Mosvold, Ball, Lai, 2016; Son & Lee, 2016). Math methods coursework has been shown to improve the CK and PCK of teacher candidates and the ability to lead discussions (Cardetti & Truxaw, 2014; Goodson-Epsy et al., 2014; Tyminski et al., 2014).

An ambitious vision for mathematics education is proposed by the National Council of Teachers of Mathematics (NCTM) in its 2000 document, Principles and Standards for School Mathematics (PSSM):

> *Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need them. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding (NCTM, 2000, p. 3).*

In reality, the mathematics education described by NCTM is quite different from the mathematics education experienced by many, if not most, of our children. In fact, Boest, Sleep, Ball, and Bass (2011) commented, "We are simply failing to reach reasonable standards of mathematical proficiency with most of our students."

Recent curriculum changes at the colleges of education in Ghana, which introduced the concept of integration by way of blending both content and methodology during teaching and learning, may have an effect on the mathematical preparation of pre-service teachers because the current curriculum requirements to integrate content and methodology during teaching and learning do not reflect research-based recommendations carried out in the colleges of education in Ghana. Without the opportunity to explore pre-service teachers' achievement of MKT in the context of teaching and learning in the face of the new curriculum, pre-service teachers at the colleges of education in Ghana may be missing out on opportunities to develop a deep conceptual understanding of content and instructional skills. Also, investigating the effects of teaching both content and methods in an integrated manner on pre-service teachers' MKT achievements while enrolled in the college of education may inform the future revision or otherwise of the teacher education curriculum. Therefore, it is the goal of this study to investigate pre-service teachers' achievement of MKT when they are taught both content and methods in an integrated format as against when they were taught content and methods separately at the colleges of education in Ghana.

The study was designed to address the following research question and hypotheses:

- 1: What is the difference between the post-test scores of PSTs who were taught Math content and methods separately and those who were taught with an integrated approach?
- H_{o1}: There is no significant difference in terms of MKT between post-test scores of PSTs who were taught math content and methods separately and those who were taught with an integrated approach.
- H_{a1}: There is a significant difference in terms of MKT between post-test scores of PSTs who were taught math content and methods separately and those who were taught with an integrated approach.

2. Literature Review

2.1. Conceptual Framework

The conceptual framework for this study focused on the development of MKT in pre-service teachers. MKT is commonly used to describe the different types and development of knowledge for in-service and PSTs of mathematics (Jenlink, 2016; Kang, 2016; Wilson, Sztajn, Edgington, & Confrey, 2014). Shulman (1987) first proposed different categories of teacher knowledge that included knowledge of content, pedagogy, curriculum, learners, and educational contexts. Ball et al. (2008) refined Shulman's model and proposed the MKT framework as a construct to conceptualize mathematical knowledge specific to the discipline of teaching. Figure 1 shows the domains of MKT and their relationship to Shulman's categories of subject matter knowledge and pedagogical content knowledge.

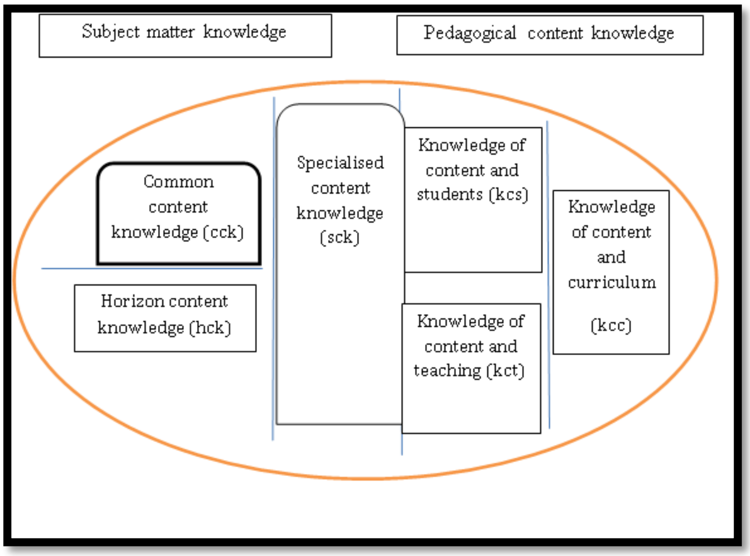

Figure 1: Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)

2.1.1. Nature and Composition of Mathematical Knowledge for Teaching

Current studies continue to probe ideas about the nature and composition of teaching-specific knowledge of mathematics. Some studies consider the construct in broad terms. They may identify or elaborate aspects or frameworks, characteristics or critique the construct, compare different representations or sub-domains, or such knowledge with other kinds of mathematical knowledge. Others examine a constrained area of knowledge: some in relation to specific mathematical topics, some in relation to specific practices of teaching or at specific levels (such as interpreting and responding to student thinking, curriculum use, or proving in high school geometry), and some in relation to specific qualities (such as connectedness). The one avenue of work representing progress in the field is the development of instruments. Instruments provide a crucial tool for investigating the nature and composition of mathematical knowledge needed for teaching. They serve to operationalize ideas about mathematical knowledge for teaching and test assumed models of the role it plays. They are used to investigate the teaching and learning of such knowledge, relationships with other variables, and other questions important for practice and policy.

The subject matter domains represent the complexity of CK necessary to teach mathematics. Ball et al. (2008) defined common content knowledge (CCK) as "the mathematical knowledge and skill used in settings other than teaching" (p. 399). CCK measures an individual's ability to obtain or recognize correct answers to math problems. Ball et al. identified knowledge that extended beyond obtaining correct solutions as specialized content knowledge (SCK). SCK is defined as "the mathematical knowledge and skill unique to teaching" (p. 400). SCK highlights the work teachers do while identifying student errors or evaluating the merit of a student's approach to a problem. Lastly, Ball et al. recognized horizon content knowledge as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). Horizon knowledge is useful in helping teachers understand the mathematical foundation they are setting with their students and what pedagogical approaches may assist in allowing students to build upon their knowledge in future learning experiences.

The pedagogical domains represent a teacher's ability to blend their knowledge of mathematics and instruction to advance students' understanding of mathematics. Ball et al. (2008) defined knowledge of content and students (KCS) as "knowledge that combines knowing about students and knowing about mathematics" (p. 401). KCS is represented in a teacher's ability to identify mathematical tasks that students will find interesting and anticipate common errors students are most likely to make. Ball et al. described the knowledge of content and teaching (KCT) as the combination of "knowing about teaching and knowing about mathematics" (p. 401). KCT is the knowledge teachers use to design instruction focusing on the impact of student learning. Lastly, Ball et al. included Shulman's (1987) category of curricula knowledge but expressed that the domain was currently unclear and may comprise a portion of the KCT domain. Investigating MKT achievements across the subdomains may be useful in understanding how MKT develops for prospective teachers enrolled in the math program using the new curriculum introduced in the Colleges of Education in Ghana, which demands that both content and methods be taught in an integrated form.

Ball et al.'s (2008) theory applies to this study by addressing my research question, which seeks to find out the extent to which PSTs develop their MKT when they are taught both content and methods in an integrated form.

Successful teaching necessitates more than mastery of the subject area. CK and PCK are distinct yet essential components of a teacher's MKT (Kleickmann et al., 2015). They asserted that CK is independent of PCK, but PCK is dependent on CK. The argument that PCK is dependent on CK is reflected in the MKT model with a joint partnership between CK and PCK. CK without PCK fails to support quality teaching (Bartell et al., 2013; Boerst, Sleep, Ball, & Bass, 2011; Baki & Arslan, 2016). Likewise, extensive evidence exists of the positive influence teachers' MKT has on student achievement (Baki & Arslan, 2016; Leong et al., 2015; Shirvani, 2015). Educational leaders are aware of the need to develop teachers' CK and PCK, which is reflected in current calls to action in the mathematics education field.

The current positions of the National Council of Teachers of Mathematics (NCTM, 2014) require students to develop conceptual understanding through investigations and discussions. However, teacher training has failed to develop skills for teaching and learning mathematics in this manner (Kaya & Aydin, 2016; Kosko, 2016; Teuscher, Moore, & Carlson, 2015). The Conference Board of the Mathematical Sciences (2012) maintained that math coursework for prospective teachers should focus on the math they will teach from a teacher's perspective. Additional recommendations included math content courses that engage PSTs in demanding math tasks, collaboration, and discourse focused on reasoning and reflection (McGalliard & Wilson, 2017).

2.1.2. Developing Teachers' Mathematical Knowledge for Teaching

With a growing sense of the importance of Mathematics for improving teaching and learning, practitioners have turned their attention to increasing teachers' knowledge of professionally relevant mathematics, and scholarly work has followed suit. A large number of studies made it clear that the design and evaluation of teacher education and professional development programs in developing teachers' mathematical knowledge for teaching are top priorities. From several decades of research, it is evident that providing teachers with opportunities to learn mathematics that are supported with instructional strategies increases their mathematical knowledge for teaching. The focus of the content, tasks, and pedagogy for teaching such knowledge requires thoughtful attention to ways of maintaining coordination of content and teaching without slipping exclusively into one domain or the other. These lessons are rooted in early efforts to document the effects of teachers' mathematical knowledge on student learning and are reinforced by current research on the design and implementation of teacher education and professional development. Much of the impetus for the surge in research on teaching-specific knowledge began with reviews of several decades of large-scale research that found surprisingly little to no effect of teachers' mathematical knowledge on their students' learning. Taking Shulman's (1986) suggestion that the content knowledge needed by teachers was characteristically different from that needed by other professionals, researchers began to look more closely at the measures used in those studies and the findings. The clearest finding that emerged was that methods courses consistently showed positive effects while content courses did not. The second was that positive effects were more likely when the content taught to teachers was more closely related to the content they subsequently taught, likewise when using student exams to measure teachers' knowledge. In general, when the mathematics taught or measured is meaningfully connected to classroom materials or interactions, it is modestly associated with improved teaching and learning. For some practitioners and policy-makers, the implication of these empirical studies, combined with logical arguments for teaching specific professional knowledge, has been enough to lead to prioritizing mathematical knowledge for teaching in the mathematical education of teachers. Nevertheless, many policies continue to press for increases in the number of mathematics courses required of teachers, regardless of their connection to teaching, despite abundant evidence that such policies are unlikely to improve teaching and learning (Yee Lai & Clark, 2018). Such policies have probably been less the result of lingering doubt about empirical results and the result of overextending the notion that knowing content well is key to good teaching, even in the face of disconfirming evidence. Of course, a certain threshold level of knowledge of the subject is essential. However, preparing teachers by requiring mathematics courses that are not directly connected to the content being taught or to the work involved in teaching that content is misguided. More recent studies continue to reinforce these established lessons. One recent line of inquiry is the investigation of features of innovative, well-received professional development programs.

The most compelling result emerging from these studies is that professional development requires designing pedagogically relevant movement between mathematical and pedagogical concerns both to motivate teachers' investment in mathematical issues and to keep the mathematical attention on mathematics that matters for the work of teaching. With deep regard for the limited effects of decades of substantial national investment in professional development, several research groups have organically developed approaches informed by thoughtful reflection and attention to the disciplined observation of teachers' engagement with and actual uptake of ideas and practices. One important insight emerging from these decades-long investments is that cycling through mathematical considerations, pedagogical considerations, and reflective enactment is vital to the design of professional development. For instance, Silverman and Thompson (2008) set out to provide evidence for whether and how teachers might enhance their mathematical knowledge for teaching through monthly practice-based professional development workshops designed to cycle from activities of doing mathematics, to examining case-based pedagogical and student-related issues, to planning, teaching and debriefing lessons collaboratively (all related to a common mathematics task or set of tasks). Examining the interactions of one teacher, they document ways these activities provided opportunities for teachers to build connections among mathematical ideas and to consider these ideas in relation to student thinking and teaching. They do not measure teacher learning, nor do they disentangle the effects of what they refer to as a professional-learning-task cycle from a number of other important features of their professional development program. However, they document dynamics in which the teacher, from an initial experience solving a nontrivial mathematics problem supported by mathematically sensitive facilitation, successively engages in mathematical issues and pedagogical issues in ways that visibly build connections among mathematical ideas, pedagogical practice, and growing mathematical knowledge for teaching. In addition, they argue that their cyclic design increased teachers' motivation for learning mathematics, both in the workshops and in their daily practice. Through successive opportunities to consider mathematical ideas in relation to the activities of classroom practice, our participants came to see their pedagogical work as infused by mathematical considerations. Similarly, in working to close the gap between a reform vision and the actual practice of mathematics teaching and learning, Kang (2016) implemented a model of professional development designed to help teachers deepen their mathematical knowledge for teaching through a cycle of solving a mathematics problem, teaching the problem, and analyzing first teacher questioning and then student thinking in order to understand the learning opportunities afforded by what they refer to as a problem-solving-cycle design. The researchers used the knowledge domains identified in Ball, Thames, and Phelps (2008) to analyze several teacher interactions. They found that different learning opportunities were afforded by different activities: specialized content knowledge was developed by comparing, reasoning, and making connections between the various solution strategies; knowledge of content and teaching was developed by analyzing teacher questioning in the video clips from the teachers' lessons; and knowledge of content and students was developed by analyzing students' solution methods (interpreting them and considering their implications for instruction). More importantly, the researchers found that reflecting on and discussing the nature of student thinking and teacher questioning of students evident in videos of their own teaching led teachers to extend their mathematical knowledge for teaching as they re-engaged with the mathematics problem and reconsidered how they might teach the problem in light of their new regard for how students might approach the problem. Throughout the analysis, the authors found that specialized content knowledge interacts with pedagogical content knowledge in interpreting student thinking and planning lessons. The authors argue that the workshops developed teachers' mathematical knowledge for teaching by supporting teachers' current knowledge while gradually challenging them to gain new understanding for the purpose of their work as teachers. Again, the dynamics between mathematics and the motivation and use of that mathematics are key to effective teacher learning of professionally relevant mathematics. The field is also beginning to see evidence that these insights have measurable yield. For instance, Bell, Wilson, Higgins, and McCoach (2010) argue that it is the practice-based character of the nationally disseminated Developing Mathematical Ideas (DMIs) and mathematics PD program that best explains participating teachers' learning of mathematical knowledge for teaching. The researchers examined pre- and post-teacher content knowledge for 308 treatment and comparison teachers across 10 well-established sites. They found significantly larger gains for treatment teachers' scores and that these gains were related to the breadth of opportunity to learn provided by facilitators. This anecdotal evidence aligns with results from S. Cohen's (2004) yearlong study of changes in teachers' thinking and practices over the course of their participation in DMI seminars (Bell et al., 2010, p. 505). These different studies compellingly add to the arguments that teachers need mathematical knowledge that is connected to the work they do and that situating the learning of mathematical knowledge in teachers' practice supports the learning of mathematical knowledge for teaching. Bell et al.'s (2010) large-scale study of the effect of professional development on teacher learning corroborates the findings of the other studies (Copur-Gencturk & Lubienski, 2013). The professional development models highlighted set teachers up to learn in and from their practice. Together, the studies discussed above point to the coordinated nature of mathematical knowledge for teaching and the ways in which the coordination between mathematics and pedagogy is essential to teaching and learning mathematical knowledge for teaching.

2.1.3. MKT Facilitating Students' Thinking, Discussion, Achievement, Engagement, and Practice

Teachers who lack high MKT, specifically in the KCT domain, may struggle to plan effective lessons (Linder & Simpson, 2017) and may not have the ability to adjust curriculum or instruction to meet the needs of their students (Lui & Bonner, 2016). Hill and Chin (2018) argued that a teacher's understanding of student thinking is positively related to students' mathematical achievement, and teachers who have a better understanding of students' thinking are more likely to adjust instruction in a manner to support students' achievement. The ability to understand student thinking and make appropriate instructional changes relates directly to the subdomains of KCS and KCT. Teachers must have the ability to develop a conceptual understanding of mathematical topics through the use of small and whole-group discussions (Tyminski et al., 2014). Ball et al.'s (2008) KCT domain, considered with Silverman and Thompson's (2008) fourth stage of MKT development, connects a teacher's ability to select activities and facilitate discussions that advance students' thinking. Also, the CCSS for mathematics and the NCTM encourages teachers to use discussion to develop students' conceptual understanding of mathematical ideas. There are benefits in terms of peer discussion and the use of probing questions by a teacher to develop the conceptual understanding of mathematical ideas in students. If teachers have low MKT levels, their students may not benefit from opportunities to develop a conceptual understanding of mathematical ideas through discussion or exploration of their peers' ideas (Kaya & Aydin, 2016; Kosko, 2016). To identify, design, and sequence learning tasks that allow students to actively engage with mathematical ideas, teachers must have unique knowledge, specifically KCS and KCT (Ball et al., 2008, Gningue, Peach, & Schroder, 2013). Chapman (2013) pointed out that teachers approach the selection of tasks differently, have differing views on the value of active learning tasks, and may benefit from additional training. Likewise, Olson and Knott (2013) maintain that teachers' perceptions of the teaching process influence how they engage students in learning. After additional training on using student-centred tasks, teachers' beliefs change toward active learning. However lasting impact on teacher practices is unclear (Polly, Neale, Pugalee, 2016). Franke et al. (2015) identified challenges students struggle with when engaging with the mathematical ideas of their peers and the complexity of a teacher's decision-making process necessary to support students' engagement. Clearly, student engagement is critical for learning, and a teacher's depth of MKT may limit their ability to engage students actively.

2.1.4. Call for Additional Research on MKT Development

There is a common recommendation to address content and pedagogy during the mathematical preparation of teachers (McGalliard & Wilson, 2017; Zambak & Tyminski, 2017). The recommendation to address content and pedagogy is supported by evidence that blending the exploration of content and instructional practices improves MKT (Auslander et al., 2016; Hoover et al., 2016; Son & Lee, 2016). Rhine (2016) also pointed out that KCS is the domain that most distinguishes master teachers from novice teachers. Similarly, Edelman (2017) stressed that prospective teachers need more instruction to develop knowledge of students, teaching, and curriculum. It is clear that CK is necessary to support teaching. Fernandez (2014) pointed out that teacher preparation programs must have a clear plan on how to build PCK. The most common method teacher preparation programs address mathematics CK, and PCK is through a combination of content, special content, and methods coursework. However, more research is necessary to identify the best practices to develop MKT within those courses. Recently, a working group conducted a thorough review of research from 1978 to 2012, focusing on prospective elementary teachers' mathematical CK. Common themes across many of the subgroups were a lack of research studies describing how MKT develops, how to improve MKT, and best practices in preparing future educators (Strand & Mills, 2014). Others have provided a similar conclusion with related literature reviews, emphasizing the need for a clearer picture of how to develop MKT in future teachers (Hoover et al., 2016). There does seem to be acting toward addressing the gaps in the literature. Thanheiser *et al.* (2014) provided evidence of PSTs learning and accompanying tasks for the math education community to review and refine. Also, (Tajudin, 2014) has taken on the task of defining the role of elementary mathematics educators with a focus on the development of MKT in prospective elementary teachers.

2.2. Implications

Findings from this study may lead to a positive social change in the form of curriculum revisions or otherwise aimed at developing prospective teachers' MKT and improving mathematics instruction for their future students. The resulting study may be shared with colleges of education and mathematics departments at other universities to improve curricula related to the development of pre-service teachers to improve the quality of Math instruction. The study may also be modified to provide professional development training for in-service teachers. The professional development training may be used to provide in-service teachers with skills to improve instructional practices with the goal of increasing teachers' achievement at MKT components. The role of teacher education programs is to prepare teachers for the demands of teaching. The teaching of mathematics entails a unique blend of knowledge. Coursework for prospective teachers has the potential to improve MKT, especially when content and instructional strategies are presented in a format conducive to students' learning. Therefore, this study aims to investigate PSTs' achievement of MKT when they are taught content and methods in a blended format at the colleges of education in Ghana.

3. Methodology

3.1. Research Design

The main design that was used in this study is the experimental design. The type of experimental design that was used is the quasi-experimental design since the subjects were not assigned randomly to the experimental and control groups (Creswell, 2014). It is a design used for comparing the achievements of two groups in a pre-test and/or post-test and also to determine how effective a treatment is. The specific type of quasi-experimental design that was used is the non-equivalent static-group comparison design. The post-test designs are used to evaluate the effects of some change in the environment on subsequent performance and hence can be employed to find out the effect of changes in an educational environment (Amedahe & Asamoa Gyimah, 2016).

3.2. Population

The population of the study comprised all pre-service teachers in the colleges of education from all over the country who are admitted to specialize in the teaching of Mathematics and their Mathematics tutors.

3.3. Sample and Sampling Procedure

The level 300 PSTs were chosen for both groups because they have gone through their mathematics programs successfully and are ready to depart for macro teaching. Two colleges of education in the Volta and Greater Accra regions of Ghana were selected purposively because they are specially training PSTs to teach mathematics at the Basic School level. These two colleges of Education were also mentored in the past by University of Cape Coast and are currently being mentored by University of Education, Winneba. PSTs in these colleges who are being specially trained to teach Mathematics at the Basic School levels will constitute the sample for the study. Both groups of PSTs will be post-tested after receiving instruction in Mathematics courses until they are ready to go for their macro teaching.

The PSTs of both groups will be examined as to when they are ready to go for their macro teaching to assess their achievements in terms of their mathematical knowledge for teaching and also to determine whether there is any significant difference in the achievements of both groups. The instruments (test items for practicing PSTs) adapted for this study comprised of items that have been used by the University of Education, Winneba, and the University of Cape Coast in numerous program evaluations and studies of relationships and effects and have been extensively validated and adapted nationally and internationally.

3.4. Instrument

The instruments for this study are the post-test (test items) adapted from the University of Cape Coast and the University of Education, Winneba item banks that have been used in numerous program evaluations and studies of relationships and effects and have been extensively validated and highly reliable. This is done to ensure item relevance and content validity.

3.5. Pilot Testing

The Mathematics Achievement Test items on MKT were field tested among PSTs in one of Colleges of Education in the region, which was not part of the accessible population for the study, and the inter-rater reliability (Pearson Product Moment) was found to be 0.73. This means that when the instrument is used on a similar sample, there is at least 73% chance of obtaining the same results.

3.6. Data Collection

The nature and purpose of the study were explained to the college authorities, having sought official permission. Data were collected from the two groups of PSTs, that is:

- The control group - They were taught content and methods courses separately, and
- The experimental group - They were taught both content and methods courses in an integrated format.

The two groups took an achievement test on their MKT as of the time they were ready to go on off-campus teaching practice, and the results were compared.

3.7. Data Analysis

The experimental and control groups' mean scores from the post-tests were analyzed using the t-test for independent samples. The t-test was more effective since it evaluated the difference between the mean scores of the groups, which enabled finding answers to the research question.

4. Results and Discussion

4.1. Research Question and Hypothesis

Research question 1 sought to find out whether there is any difference between post-test scores of PSTs who were taught math content and methods separately and those who were taught with an integrated approach with regard to their MKT. The hypothesis formulated and tested based on the research question is stated below.

- Ho1: There is no significant difference in terms of MKT between post-test scores of PSTs who were taught math content and methods separately and those who were taught with an integrated approach.
- Ha1: There is a significant difference in terms of MKT between post-test scores of PSTs who were taught math content and methods separately and those who were taught with an integrated approach.

Table 1 shows the means and the standard deviations of the scores obtained by the experimental and the control group on the post-test. The table reviews that the control group [N=117, M=30.43, SD=11.476] had a mean score lower than the experimental group, and the standard deviation shows that the scores of the control group were far from their mean score. That is, the scores of the control group were more heterogeneous than that of the experimental group. The experimental group [N=130, M=35.47, SD=10.454] scored higher than the control group on the post-test, and their scores were more homogeneous than the control group. To test whether the difference in the means of the control and the experimental group were significantly different, an independent samples t-test was run to compare their mean scores at a significant level of 0.05 two-tailed. Table 1 shows the results of the independent samples t-test.

	The Groups That Students Belong to	N	Mean	Std. Deviation	Std. Error Mean
Scores of Students on a Test	Control	117	30.43	11.476	1.061
	Experimental	130	35.47	10.454	0.917

Table 1: Descriptive Statistics of Post-Test Scores for Control and Experimental Groups

From table 2, Levene's test for equality of variance reveals no significant difference between the variances of the control and the experimental group [F=1.423, p > 0.05]. That is, the difference observed in the standard deviations of the two groups is not statistically significant. Since Levene's test for equality of variance revealed no significant difference between the variances of the control and the experimental group, the t-value corresponding to equal variances assumed was used to interpret the result of the independent samples t-test. The t-test revealed that at a significant level of 0.05, there is a statistically significant difference between the means of the two groups on the post-test [t = 3.613, p < 0.05]. With a mean difference of 5.042, the experimental group performed better than the control group on the post-test. Hence, the students in the experimental group turned out to achieve better on MKT components than the students in the control group. Thus, the integrated approach seemed superior to the separate approach with regard to assisting the students in developing mathematical knowledge for teaching.

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
scores of students on a test	Equal variances assumed	1.423	0.234	-3.613	245	0	-5.042	1.395	-7.79	-2.293
	Equal variances are not assumed.			-3.596	235.736	0	-5.042	1.402	-7.804	-2.279

Table 2: Results of Independent Sample t-test of the Post-Test Scores for Control and Experimental Groups

4.2. Findings and Discussions

Analysis of the post-test scores using the independent sample t-test revealed that the control group [N=117, M=30.43, SD=11.476] had a mean score lower than the experimental group, and the standard deviation shows that the scores of the control group were far from their mean score. That is, the scores of the control group were more heterogeneous than that of the experimental group. The experimental group [N=130, M=35.47, SD=10.454] scored higher than the control group on the post-test, and their scores were more homogenous than the control group.

The t-test again also showed that at a significant level of 0.05, there is a statistically significant difference between the means of the two groups on the post-test [t = 3.613, p < 0.05]. With a mean difference of 5.042, the experimental group performed better than the control group on the post-test. Hence, the students in the experimental group turned out to achieve better on MKT components than the students in the control group. This finding is in consonance with the findings of McGalliard & Wilson (2017) and Zambak & Tyminski (2017), which indicated that the students taught with the integrated approach achieved better mathematical knowledge for teaching components than the students taught with the separate approach. It also gave credence to the studies conducted by Auslander, Smith, Hart, & Carothers (2016), Hoover, Mosvold, Ball, Lai (2016), and Son & Lee (2016)**,** who found that presenting content and instructional practices in an integrated format affect the development of MKT more than addressing the concepts individually.

5. Conclusions

Based on the findings of this study, the following conclusions were drawn:

- The students in the experimental group turned out to achieve better on MKT components than the students in the control group.
- The integrated approach seemed superior to the separate approach with regard to assisting the students in developing mathematical knowledge for teaching.

6. Recommendations

The following recommendations were made for educational practice and policy:

- The use of the integrated approach in the teaching and learning of Mathematics should be encouraged and monitored throughout the Colleges of Education in Ghana.
- The Ministry of Education, in collaboration with Ghana Tertiary Education Commission (GTEC) and Transforming Teacher Education and Learning (T-TEL), should organize more in-service training for tutors in the Colleges of Education in Ghana to equip them with consolidated skills needed to handle the integrated approach to the teaching and learning of Mathematics with mastery.
- Again, the Ministry of Education, in collaboration with Ghana Education Service and other relevant stakeholders, should organize more in-service training for basic and secondary teachers already in the field to enable them to acquire the needed skills in order to effectively use the integrated approach in the teaching and learning of Mathematics.

7. References

i. Amedahe, F. K., & Asamoa Gyimah, E. (2016). Introduction to educational research (4th ed.). Kumasi, Ghana: Yaci Publications.
ii. Auslander, S. S., Smith, S. Z., Smith, M. E., Hart, L. C., & Carothers, J. (2016). A case study of two groups of elementary prospective teachers’ experiences in distinct mathematics content courses. Southeastern Regional Association of Teacher Educators, 25(2), 1–15.
iii. Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59, 389–407.
iv. Baki, M., & Arslan, S. (2016). Reflections from pre-service teachers' mathematics teaching process. Hacettepe University Journal of Education, 31(4), 736-749. doi:10.16986/HUJE.2015014664
v. Bartell, T. G., Webel, C., Bowen, B., & Dyson, N. (2013). Prospective teacher learning: Recognizing evidence of conceptual understanding. Journal of Mathematics Teacher Education, 16(1), 57–79. doi:10.1007/s10857-012-9205-4
vi. Bell, C. A., Wilson, S., Higgins, T., & McCoach, D. B. (2010). Measuring the effects of professional development on teachers’ knowledge: The case of developing mathematical ideas. Journal for research in mathematics education, 41(5), 479–512.
vii. Boerst, T., Sleep, L., Ball, D., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. Teachers College Record, 113(12), 2844–2877.
viii. Cardetti, F., & Truxaw, M. P. (2014). Toward improving the mathematics preparation of elementary pre-service teachers. School Science and Mathematics, 114, 1–9.
ix. Chapman, O. (2013). Mathematical-task knowledge for teaching. Journal of Mathematics Teacher Education, 16(1), 1–6.
x. Conference Board of the Mathematical Sciences (2012). The Mathematical Education of Teachers II. Providence, RI, and Washington DC: American Mathematical Society and Mathematical Association of America.
xi. Copur-Gencturk, Y., & Lubienski, S. T. (2013). Measuring mathematical knowledge for teaching: A longitudinal study using two measures. Journal of Mathematics Teacher Education, 16, 211–236. doi:10.1007/s10857-012-9233-0
xii. Creswell, J. W. (2014). Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (4th ed.). Los Angeles, CA: Sage.
xiii. Edelman, J. (2017). How pre-service teachers use children's literature to teach mathematical concepts: Focus on mathematical knowledge for teaching. International Electronic Journal of Elementary Education, 9(4), 741–752. Retrieved from www.iejee.com
xiv. Fauskanger, J. (2015). Challenges in measuring teachers’ knowledge. Educational Studies in Mathematics, 90, 57–73. doi:10.1007/s10649-015-9612-4
xv. Fernandez, C. (2014). Knowledge base for teaching and pedagogical content knowledge (PCK): Some useful models and implications for teachers’ training. Problems of Education in the 21st Century, 60, 79–100.
xvi. Franke, M. L., Turrou, A. C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with others’ mathematical ideas: The role of teacher invitation and support moves. The Elementary School Journal, 116, 126–148.
xvii. Gningue, S. M., Peach, R., & Schroder, B. (2013). Developing effective mathematics teaching: Assessing content and pedagogical knowledge, student-centered teaching, and student engagement. The Mathematics Enthusiast, 10(3), 621–645.
xviii. Goodson-Epsy, T., Lynch-Davis, K., Cifarelli, V. V., Morge, S., Pugalee, D., & Salinas, T. (2014). Applying NAEP to improve mathematics content and methods courses for pre-service elementary and middle school teachers. School Science and Mathematics, 114(8), 392–404.
xix. Hill, H., C., Charalambous, C. Y., & Chin, M. J. (2018). Teacher characteristics and student learning in mathematics: A comprehensive assessment. Educational Policy, 1-32. doi:10.1177/0895904818755468
xx. Hill, H. C., & Chin, M. J. (2018). Connections between teachers’ knowledge of students, instruction, and achievement outcomes. American Educational Research Journal, 55(5), 1076–1112. doi:10.3102/0002831218769614
xxi. Hill, H.C., Umland, K., Litke, E., & Kapitula, L. R. (2012). Teacher quality and quality teaching: Examining the relationship of a teacher assessment to practice. American Journal of Education, 118, 489–519.

xxii. Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching, The Mathematics Enthusiast, 13(2), 1551–3440.

xxiii. Jenlink, P. (2016). Preparing mathematics teachers with substantive knowledge. Teacher Education and Practice, 29(4), 585–594.

xxiv. Kajander, A., & Holm, J. (2016). What math matters? Types of mathematics knowledge and relationships to methods course performance. Canadian Journal of Science, Mathematics and Technology Education, 16(3), 273–283, doi: 10.1080/14926156.2016.1183837

xxv. Kang, R. (2016). Deepening novice teachers' mathematical knowledge for teaching through lesson study in a collaborative action research context. Teacher Education and Practice, 29(4), 682–698.

xxvi. Kaya, D., & Aydin, H. (2016). Elementary mathematics teachers' perceptions and lived experiences on mathematical communication. EURASIA Journal of Mathematics, Science & Technology Education, 13(6), 1619–1629. doi:10.12973/eurasia.2014.1203a

xxvii. Kleickmann, T., Richter, D., Kunter, M., Elsner, J., Besser, M., Krauss, S., Cheo, M., & Baumert, J. (2015). Content knowledge and pedagogical content knowledge in Taiwanese and German mathematics teachers. Teaching and Teacher Education, 46, 115–126. doi:10.1016/j.tate.2014.11.004

xxviii. Kosko, K. W. (2016). Primary teachers' choice of probing questions: Effects of MKT and supporting student autonomy. IEJME-Mathematics Education, 11(4), 991–1012. Retrieved from http://www.iejme.com/makale/505

xxix. Leong, K. E., Meng, C. C., & Abdul Rahim, S. S. (2015). Understanding Malaysian pre-service teachers' mathematical content knowledge and pedagogical content knowledge. Eurasia Journal of Mathematics, Science and Technology Education, 11(2), 363–370. doi:10.12973/eurasia.2015.1346a

xxx. Linder, S. M., & Simpson, A. (2017). Towards an understanding of early childhood mathematics education: A systematic review of the literature focusing on practicing and prospective teachers. Contemporary Issues in Early Childhood, 19(3), 1-23. doi:10.1177/1463949117719553

xxxi. Lui, A. M., & Bonner, S. M. (2016). Pre-service and in-service teachers' knowledge, beliefs, and instructional planning in primary school mathematics. Teaching and Teacher Education, 70, 121–131. doi:10.1016/j.tate.2017.11.013

xxxii. Maher, N., & Muir, T. (2013). "I know you have to put down a zero, but I'm not sure why": Exploring the link between pre-service teachers' content and pedagogical content knowledge. Mathematics Teacher Education and Development, 15, 72–87.

xxxiii. McGalliard, W. A., & Wilson, P. H. (2017). Examining aspects of elementary grades pre-service teachers' mathematical reasoning. Investigations in Mathematics Learning, 9(4), 187–201, doi: 10.1080/19477503.2016.1258857

xxxiv. National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM.

xxxv. National Council of Teachers of Mathematics. (2000). Principles and standards for School Mathematics, Reston, VA: NCTM.

xxxvi. Olson, J. C., & Knott, L. (2013). When a problem is more than a teacher's question. Educational Studies in Mathematics, 83, 27–36.

xxxvii. Polly, D., Neale, H., & Pugalee, D. K. (2014). How does ongoing task-focused mathematics professional development influence elementary school teachers' knowledge, beliefs, and enacted practices? Early Childhood Education Journal, 42, 1–10.

xxxviii. Rhine, S. (2016). The critical nature of the knowledge of content and students' domain of mathematical knowledge for teaching. Teacher Education and Practice, 29(4), 595–614.

xxxix. Shirvani, H. (2015). Pre-service elementary teachers' mathematics content knowledge: A predictor of sixth graders' mathematics performance. International Journal of Instruction, 8, 133–142.

xl. Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. Harvard Educational Review, 57, 1–22.

xli. Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15, 4–14

xlii. Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. Journal of Mathematics Teacher Education, 11, 499–511. doi:10.1007/s10857-008-9089-5

xliii. Smith, M. E., Swars, S. L., Smith, S. Z., Hart, L. C., & Haardorfer, R. (2012). Effects of an additional mathematics content course on elementary teachers' mathematical beliefs and knowledge for teaching. Action in Teacher Education, 34, 336–348. doi:10.1080/01626620.2012.712745

xliv. Son, J. W., & Lee, J. E. (2016). Pre-service teachers' understanding of fraction multiplication, representational knowledge, and computational skills. Mathematics Teacher Education and Development, 182, 5–28.

xlv. Strand, K., & Mills, B. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on algebra. The Mathematics Enthusiast, 11(2), 385–1551.

xlvi. Teuscher, D., Moore, K. C., & Carlson, M. P. (2015). Decentering: A construct to analyze and explain teacher actions as they relate to student thinking. Journal of Mathematics Teacher Education, 19(5), 433–456. doi:10.1007/s10857-015-93040 xlvii. Thanheiser, E., Browning, C., Edson, A. J., Lo, J. J., Whitacre, I., Olanoff, D., & Morton, C. (2014). Prospective elementary mathematics teacher content knowledge: What do we know, what do we not know, and where do we go? The Mathematics Enthusiast, 11(2), 1551–3440.

xlviii. Tyminski, A. M., Zambak, V. S., Drake, C., & Land, T. J. (2014). Using representations, decomposition, and approximations of practices to support prospective elementary mathematics teachers' practice of organizing discussions. Journal of Mathematics Teacher Education, 17, 463–487. doi:10.1007/s10857-013-9261-4

xlix. Wilson, P. H., Sztajn, P., Edgington, C., & Confrey, J. (2014). Teachers' use of their mathematical knowledge for teaching in learning a mathematics learning trajectory. Journal of Mathematics Teacher Education, 17, 149–175.

l. Yee Lai, M., & Clark, L. J. (2018). Extending the notion of Specialized Content Knowledge: Proposing constructs for SCK. Mathematics Teacher Education and Development, 20(2), 75–95.

li. Zambak, V. S., & Tyminski, A. M. (2017). A case study on specialized content knowledge development with dynamic geometry software: The analysis of influential factors and technology beliefs of three pre-service middle grades mathematics teachers. Mathematics Teacher Education and Development, 191, 82–106.