INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH & DEVELOPMENT

ISSN 2278 – 0211 (Online)

Modeling Volatility of Daily Stock Return Using Generalized Autoregressive Conditional Heteroscadatic with Skewed Error Distribution and Stochastic Volatility Models

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Abstract:
The problem of how best to model volatility of stock prices returns have continued to occupy the minds of researchers in this area. This study therefore compares the performance of two competitive volatility models: The Stochastic Volatility (SV) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) based on skewed error distributions models using daily stock returns of Fourteen Nigerian banks. The daily stock closing price of the selected banks were collected for the period 4/1/2007 to 31/12/2021 and the stationarity, normality and ARCH effect of the series were tested. The parameter of SV and different types of GARCH models based on skewed normal, skewed student t and skewed generalized error distribution were estimated and selection of the best model was done using Akaike Information Criterion (AIC). Post estimation and evaluation were carried out using the Mean Square Error (MSE) and Root Mean Square Error (RMSE). Results of analysis revealed a stationary but asymmetrically distributed return series with ARCH effect. The model forecasting performance proved APARCH (1,1) based on student t-distribution as the best among the competing GARCH models with the record of the least AIC value in ten of the fourteen daily bank stocks returns. Although, APARCH (1,1) and SV models were found to be comparable. SV model was however recommended as the best since it recorded the least RMSE value in 11 (79%) of the 14 bank stocks against 3 (21%) bank stock in favour of APARCH (1,1). This implies that SV model was found to be adequate in modeling volatility of the daily stock returns of ECO Bank, First City Monument bank, Fidelity bank, first bank, Guaranty Trust bank, Stanbic IBTC bank, Sterling bank, United Bank for Africa, Union bank, WEMA bank and Zenith bank; while APARCH (1,1) base on Skewed student 't' distribution was preferred in modeling volatility of Access bank, Skye Bank and Unity bank. Results also showed the presence of volatility persistence, suggesting uncertainties and the risk of losses by investors. Therefore, it can be deduced that the application of a stochastic volatility model to the selected stock returns would drastically minimize the losses by investors.

Keywords: *Volatility, stochastic volatility, Garch, Aparch, skewed error distribution, volatility persistence, leverage effect*

1. Introduction

Time-varying volatility models have been extensively used to analyze time series data. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and the Stochastic Volatility (SV) models are major and competitive volatility models. They are well-known in financial data analysis. According to Jouchi (2009), these models and their extensions capture the pattern and trend of data, especially in finance, like stock prices.

Bollerslev (1986) first suggested that the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is defined as successive dependence of volatility and combined prior observations into the future volatility.

Nelson (1991) formulates the Exponential Generalized Autoregressive Conditional Heteroskedastic (E-GARCH) specification. It is aimed at addressing the leverage effect in the volatility. On the other hand, the GARCH model with leverage effect had been treated differently by Glosten, Jagannathan, and Runkle (1991) as the Glosten, Jagannathan, and Runkle Generalized Autoregressive Conditional Heteroskedasticity (GJR-GARCH) model and this became a standard for the Asymmetric Power Autoregressive Conditional Heteroskedasticity (APARCH) model. Conversely, Stochastic Volatility (SV) models have been studied and established to be a continuous-time probability process. It is severally applied in financial econometrics as submitted by Ghysels, Harvey, and Renault (1995). The volatility variance is assumed to be non-linear, with an additional error term which makes the model more flexible. In the word of Francq and Zakoian (2010), Modeling return volatility of the stock over a given period of time seems very complex and difficult as the data series can often be viewed using different frequencies of observation; this may be in seconds, minutes, hours, days, months and years. One of the distinguishing features of stock volatility is that it reveals an element of risk or uncertainty, Tsay (2005). The volatility of an asset can simply measure this risk or uncertainty. The problem often encountered while modeling stock volatility is the concept of non-stationarity, Bruce (2011). Non-stationarity occurs when the underlying rules that generate the time series change occasionally, often without any indication that a change is about to happen. According to Sherry and Sherry (2000), while dealing with a non-stationary time series data, one is essentially dealing with a high level of uncertainty. Therefore, there is a maximum risk to investment.

The GARCH family of models is observation-driven, whereas the Stochastic Volatility models are parameter-driven. Apart from the presence of heteroscedasticity, some of the motivations for using the GARCH family of models are that time series of financial asset returns often exhibit volatility clustering and fat tails or leptokurtosis. Here, the current volatility is strongly related to the volatility present in the immediate past, Brooks (2008); Francq and Zakoian (2010). Leptokurtosis occurs when the distribution of the return of an asset exhibits fatter tails and is more peaked at zero than that of a standard normal distribution. Another reason for using the GARCH family of models is that financial time series often exhibit a leverage effect, which is an asymmetry of the impact of past positive and negative values on the current volatility. It is often seen that negative returns (a price decrease) tend to increase the volatility by a larger amount than a positive return (price increase) of the same amount Francq and Zakoian (2010).

The Stochastic Volatility models have not been used as widely as the GARCH family of models. One of the reasons for this is that the likelihood for the Stochastic Volatility models is not easy to evaluate, which is not the case with the GARCH models, Shimada and Tsukuda (2005). There are two reasons for the difficulty in estimating the likelihood for Stochastic Volatility models.

- Firstly, the variance is modeled as an unobserved component.
- Secondly, the model is non-Gaussian.

This results in complicated likelihood estimation, Mahieu, and Schotman (1998).

1.1. Statement of Problem

The study of stock price volatility has gained popularity over the years due to its importance in both financial markets and government planning. However, several studies have been conducted on understanding and modeling stock price volatility. This includes, among others:

- A research to examine the stock returns volatility in view of the global financial crisis in Nigeria, Olowe (2009);
- Modeling volatility of four Nigerian firms using GARCH model by Onwukwe, Bassey, and Isaac (2011),
- Modeling and forecasting of volatility pattern of daily returns of fifteen (15) Nigerian Commercial banks stocks by Onwukwe, Samson, and Lipsey (2014).
- Finally, the study concluded that E-GARCH (1,1) model was the appropriately forecasting model among other works.

However, GARCH model considered in those studies is a deterministic model. This is in contrast to Brooks (2008), who showed that stock price volatility is in itself stochastic and skewed.

The work of Alberg, Shalit, and Zhang (2009) showed that using error innovation with skewed parameters improved fitness and forecasting performance of volatility. The findings of Samson, Onwukwe, and Enang (2020) that in modeling volatility without using error distribution with a skewed parameter may not give a true estimate of the actual volatility in the financial time series. Also, a few studies, including Kim et al. (1998), Giot and Laurent (2004), examined the model comparisons among the models GARCH and SV models. However, none has compared the forecasting performance of GARCH based on skewed innovations with SV models. In view of the preceding, it is evident that an attempt to model volatility with GARCH without considering various skewed innovations may be inappropriate, misrepresented, and misleading. Therefore, this study seeks to model the volatility of daily stock returns of fourteen (14) Nigerian banks using GARCH model based on the skewed error distribution model and Stochastic Volatility model and further compare the forecasting performance to determine the most appropriate model. The success of this work would further present a more appropriate and reliable volatility model. The model would help financial planners, investors, risk managers, economists, policymakers, government agencies, corporate organizations, and researchers to plan and implement policies and research to improve the profit margin and strengthen the national economy.

2. Methods

2.1. Study Data

This study used daily closing stock price data of fourteen (14) selected Nigerian banks, namely: Access Bank, ECO Bank, First City Monument Bank (FCMB), Fidelity, First Bank, Guaranty Trust Bank, Stanbic IBTC bank, Skye Bank, United Bank for Africa (UBA), Unity Bank, WEMA Bank, Union Bank, Zenith Bank. These banks are listed on the Nigerian Stock Exchange of Commercial Banks License with International and National Authorization to model its volatility. The data were collected on a daily basis for Fourteen (14) years starting from 4th January, 2007 to 31st December, 2021 from www.cashcraft.com. We use data from 4th January, 2007 to December, 31st 2019 for model formation and data from 4th January 2021 to 21st December 2021 (out-of-sample) for model forecast performance.

2.2. Computations of Return Series from Price

The daily logarithm returns were calculated from the logarithm of the daily closing price, which is given by:

$$r_t = In\left(\frac{p_t}{p_{t-1}}\right), \text{t} = 2,3, \dots, \text{T} \quad (1)$$

Where p_1 and p_{t-1} are the closing prices at times t and $t-1$, respectively. T is the number of observations (Ruppert, (2004).

2.3. Normality of the Return Series

This test assesses whether the returns residual of the chosen stocks are normally distributed. We test for normality of the series using the Jacque Bera (B) test denoted by:

$B = \frac{n}{6}\left[\rho^2 + \frac{(\eta-3)^2}{4}\right]$ (2)

Where, ρ is the skewness and η is the kurtosis. If $\eta<0$, it is platykurtic; if $\eta > 0$, it is excess kurtosis, and if $\eta=0$, it is mesokurtic. $B \sim \chi_2^2$ and the null hypothesis is rejected if the p< 0.05

2.4. Test for Stationarity of Return Series

The Augmented Dickey-Fuller Test (ADF) was used to test the stationarity of the daily returns series. Given null and alternative hypotheses as

$H_0: \theta = 1$ and $H_1: \theta < 1$ respectively

The test Statistic:

t-ratio = $\frac{\hat{\theta}-1}{Std(\theta)} = \frac{\sum_{r=2}^{T} P_{t-1}e_t}{\hat{\sigma}^2\sqrt{\sum_{r=2}^{T} P_{t-1}^2}}$ (3)

where $\hat{\theta} = \frac{\sum_{r=2}^{T} P_{t-1}P_t}{\sqrt{\sum_{r=2}^{T} P_{t-1}^2}}$ (4)

and $\hat{\sigma}^2 = \frac{\left(\sum_{r=2}^{T} P_t - \hat{\theta}P_{t-1}\right)^2}{T-1}$ (5)

T is the total number of observations of the returns. The H_0 is rejected if $P < 0.05$

2.5. Heteroscedasticity Test

The Lagrange multiplier test (LMT) statistic was applied to the series to check for heteroscedasticity (ARCH effect). The conditional heteroscedasticity can be expressed as:

$a_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_n a_{t-n}^2 + \varepsilon_t, t = n+1, \ldots, T$ (6)

The hypotheses to be tested are:

$H_o: \alpha_1 = \cdots = \cdots \alpha_n = 0$ vs $H_1: \alpha_t \neq 0$ for some $i \in \{1,2,\ldots,m\}$

Test statistic

$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1(T-2m-1)}$ (7)

where

$SSR_0 = \sum_{t=m+1}^{T}(a_t^2 - \bar{\omega})^2; SSR_1 = \sum_{t=m+1}^{T}\hat{e}_t^2; \bar{\omega} = \frac{1}{T}\sum_{t=1}^{T} a_t^2$

Where $\hat{e}_t^2$ is a square error (8), SSR_0 is the total sum of squares, and $\bar{\omega}$ is the sample mean of a_t^2. The test statistic is assumed to follow the chi-square distribution with m degree of freedom. Therefore, we reject $H_o if F > \chi_m^2(\alpha)$, where $\chi_m^2(\alpha)$ is the upper $100(1-\alpha)$ percentile of chi-squared distribution with m degrees of freedom, or if the p-value of F is less than 0.05, and T is the number of observation Tsay (2005).

2.6. Volatility Models Considered in the Study

2.6.1. GARCH Models and Its Extension

2.6.1.1. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Models

The GARCH (p,q) model is given by

$r_t = \mu + a_t, \quad \sigma_t^2 = \alpha_0 + \alpha_i a_{t-i}^2 + \beta_j \sigma_{t-j}^2 \quad ; a_t = \sigma_t \varepsilon_t$ (8)

Where ε_t~N(0,1). α_0, α_i, $\beta_j \geq 0$ for σ_t^2 to be positive and for stationarity $\alpha_i + \beta_j < 1$; α_i is ARCH term while β_j is GARCH term, σ_t is volatility, r_t is returns and a_t is the residuals.

Thus, $\alpha_i + \beta_j < 1$ for variance to be finite

2.6.1.2. Integrated Generalized Autoregressive Conditional Heteroscedasticity (I-GARCH) Model

The integrated GARCH (I-GARCH) process was designed to model data that exhibit persistent changes in volatility.

The IGARCH model is given as:

$r_t = \mu + a_t, \quad \sigma_t^2 = \alpha_0 + \alpha_i a_{t-i}^2 + \beta_j \sigma_{t-j}^2 \quad ; a_t = \sigma_t \varepsilon_t$ (9)

Where α_0 is a constant, $\alpha_i$is ARCH term while β_j is GARCH term, σ_t is volatility, r_t is returns, and a_t is the residuals. The GARCH (p, q) process is called I-GARCH if

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1 \quad (10)$$

This is the stationary condition for I-GARCH (Ruppert, 2004).

2.6.1.3. Exponential Generalized Autoregressive Conditional Heteroscedasticity (E-Garch)

The Exponential GARCH (E-GARCH) model was first introduced by (Nelson, 1991). The model allows for asymmetric effects between positive and negative asset returns.

An E-GARCH (p,q) model can be written as:

$$r_t = \mu + a_t, \quad In(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(\theta a_{t-i} + \gamma_i \left[|a_{t-i}| - \sqrt{\frac{2}{\pi}}\right]\right) + \sum_{j=1}^{q} \beta In(\sigma_{t-j}^2) \quad (11)$$

where α_0 is a constant, α_i is ARCH term while β_j is GARCH term, γ is the leverage term, σ_t is volatility, r_t is returns, and a_t is the residuals. It is strictly stationary if and only if $|\beta| < 1$

2.6.1.4. Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH)

According to Rossi (2004), the asymmetric power ARCH model, proposed by Ding et al. (1993), given below, forms the basis for deriving the GARCH family models.

$$r = \mu + a_t, \ \ \sigma_t^{\delta} = \alpha_0 + \left(\sum_{i=1}^{p} \alpha_i(|a_{t-i}|) - \gamma_i a_{t-i}\right)^{\delta} + \sum_{j=1}^{q} \beta_j \left(\sigma_{t-j}^2\right)^{\delta} \qquad (12)$$

Where $\alpha_0 > 0,\ \delta \geq 0.\ \ \alpha_i \geq 0,\ i = 1,2 \ldots, p-1,\ i = 1,2,\ldots,p;\ \ \beta_j \geq 0, j = 1,2,\ldots,q-1$ and $-1 < \gamma_i < 1$. If p=1, we have APARCH (1,1)

2.6.1.5. Threshold Generalized Autoregressive Conditional Heteroscedasticity (T-GARCH) Model

The Threshold GARCH model is another model used to handle leverage effects; it was developed by Glosten et al. (1993). T-GARCH (p,q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad (13)$$

where N_{t-i} is an indicator for negative a_{t-i}; that is,

$$N_{t-i} = \begin{cases} 1 \ if a_{t-i} < 0 \\ 0 \ if a_{t-i} \geq 0 \end{cases}$$

Furthermore, a_t, γ_i and β_j are non-negative parameters satisfying conditions similar to those of GARCH models (Tsay, 2005). When $p = 1, q = 1$, the T-GARCH (1,1) model becomes:

$$\sigma_t^2 = \alpha_0 + (\alpha_i + \gamma_i N_{t-i})a_{t-i}^2 + \beta_j \sigma_{t-j}^2 \qquad (14)$$

2.6.1.6. Gloseten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroscedasticity (GJR-GARCH) Model

The GJR-GARCH (p,q) Model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$. when $\delta = 2$ and $0 \leq \gamma_i < 1$,

If p=1, and q=1, we have GJR_GARCH (1,1) as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i(|a_{t-i}| - \gamma_i a_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \ ; \qquad a_t = \sigma_t \varepsilon_t \qquad (15)$$

<u>2.6.2. Skewed Distribution</u>

2.6.2.1. The Skewed Normal Distribution Has the Pdf

$$f(\varepsilon_t) = \frac{1}{\theta\pi} e^{\frac{(\varepsilon_t - \vartheta)^2}{2\theta^2}} \int_{-\infty}^{\beta} \frac{\varepsilon_t - \vartheta}{\theta} \ e^{\frac{t^2}{2}} \ dt, \qquad -\infty < \varepsilon_t < \infty \qquad (16)$$

where ϑ is the location parameter, θ is the scale, and β is the shape parameter.

2.6.2.2. Skewed Students' Distribution

The Skewed Student 't' Distribution has the pdf.

$$f(\varepsilon_t, \mu, \vartheta, \emptyset, \lambda) = \begin{cases} bc\left[1 + \frac{1}{\phi - 2}\left(\frac{b\left(\frac{\sigma_t - \mu}{\vartheta}\right) + a}{1 - \lambda}\right)^2\right]^{-\frac{\rho+1}{2}} & \varepsilon_t < -\frac{a}{b} \\ bc\left[1 + \frac{1}{\phi - 2}\left(\frac{b\left(\frac{\sigma_t - \mu}{\vartheta}\right) + a}{1 + \lambda}\right)^2\right]^{-\frac{\rho+1}{2}} & \varepsilon_t \geq -\frac{a}{b} \end{cases}, \qquad (17)$$

Where $\emptyset$ and λ represent the shape and skewness parameters respectively

$\boldsymbol{a} = \boldsymbol{4}\lambda c\left(\frac{\emptyset-2}{\emptyset-1}\right), b = 1 + 3\lambda^2 - a^2, \boldsymbol{c} = \frac{\Gamma\left(\frac{\emptyset+1}{2}\right)}{\sqrt{\pi(\emptyset-2)\Gamma\left(\frac{\emptyset}{2}\right)}}$

2.6.2.3. Skewed Generalized Error Distribution

$$f(\varepsilon_t / \rho, \boldsymbol{\omega}, \theta, \delta) = \frac{\rho}{2\theta\Gamma\left(\frac{1}{\rho}\right)} \ exp \ \left[-\frac{|\varepsilon_t - \delta|^{\rho}}{[1 + sign(\varepsilon_t - \delta)a]^{\rho\theta^{\rho}}}\right] \qquad (18)$$

$\theta > 0, -\infty < \varepsilon_t < \infty, \qquad \rho > 0, -1 < \boldsymbol{\omega} < \boldsymbol{1}, \ -\propto < \delta < \infty$

$\theta = \Gamma\left(\frac{1}{\rho}\right)^{0.5} \Gamma\left(\frac{3}{v}\right)^{-0.5} S(\omega)^{-1}, \delta = 2\omega S(\omega)^{-1}, S(\omega) = \sqrt{1 + 3\omega^2 - 4A^2\omega^2}, \qquad A = \Gamma\left(\frac{2}{\rho}\right)\Gamma\left(\frac{1}{\rho}\right)^{-0.5} \Gamma\left(\frac{3}{\rho}\right)^{-0.5}$

Where $\rho > 0$ is parameter shape, ω is skewness parameter with $-1 < a > 1$

<u>2.6.3. Stochastic Volatility (SV) Model</u>

Kim, Shephard, and Chib (1998) represented SV as:

$$r_t = \beta e^{\frac{h_t}{2}} a_t \qquad t \geq 1 \qquad (19)$$

$$h_t = \mu + \varphi(h_{t-1} - \mu) + \sigma_{\varepsilon}\varepsilon_t \qquad (20)$$

where r_t is the mean corrected return at time t, h_t is the log-volatility at time t, which is assumed to follow a stationary process ($|\varphi| < 1$) with h_t drawn from a stationary distribution, a_t and ε_t are uncorrelated standard normal white noise shocks, and σ_ε are the volatility of the log-volatility. The parameter β or $e^{\frac{\mu}{2}}$ is a constant scaling factor and can be considered modal instantaneous volatility. In some cases, μ is taken as0, so β will be 1. φ is the persistence in the volatility.

Note that the estimation of the process of SV is not directly observable. Therefore, an addition likelihood function must be constructed to include the behavior of the collected data. Jaquier et al. (1994) propose a Bayesian approach, using the Monte Carlo Markov Chain (MCMC) technique where the posterior distribution of the parameter is sampled. The parameter space is (θ, h) where $\theta = (\varphi, \sigma_\varepsilon^2, \mu)$, and the sampling algorithm is given in Kim, Shephard, and Chip (1998) as follows:

1. Initialize h and $\theta.$
2. Sample h_t from $h_{-t}, r, \theta,\ t = 1, \dots, T$
3. Sample $\sigma_\varepsilon^2|r, h, \varphi, \mu$
4. Sample $\varphi|r, h, \mu, \sigma_\varepsilon^2$
5. Sample $\mu|r, h, \varphi,\ \sigma_\varepsilon^2$
6. Go to 2.

Cycling from 2 to 5 is a complete sweep of this sampler. Many sweeps should be performed to generate samples from $\theta, h|r$

Kim, Shephard, and Chip (1998) stated all of the parameters mentioned above explicitly.

2.7. Estimation of the Parameter

The parameters of these volatility models are estimated using Rugarch package and Stocvol in R programme.

2.8. Model Diagnostic and Selection Criterion

Akaike Information Criterions (AIC) is used in model selection criteria which are given as:

$$\text{AIC} = 2\beta - 2\ln(\text{LL}) = 2\beta + 2\ln\left(\frac{\text{RSS}}{\text{n}}\right) \tag{21}$$

Where, RSS=$\sum_{i=1}^{T} \hat{e}^2$ is the residual sum of squares

LL is the maximized value of the log-likelihood for the estimated volatility model, and β is the number of independent parameters in the model.

This technique selects the model with the least value of AIC. It is computed as:

2.9. Model Forecasting Performance

The Root Mean Squared Error (RMSE) statistic is used for model forecasting. The Statistic is given as:

$$\text{RMSE} = \frac{1}{\text{T}-1}\sqrt{\sum_{t=2}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2} \tag{22}$$

2.10. Comparison of GARCH and Stochastic Volatility Models

The GARCH model for each of the bank returns would be compared with the Stochastic Volatility model estimated using RMSE. The model with the least RMSE was considered appropriate and fitted the respective bank stocks.

3. Results

3.1. Descriptive Statistics

The descriptive statistic of the daily stock returns data of the fourteen (14) selected bank stocks are presented in table 1. The results showed that the mean return series for all the banks except Stanbic IBTC bank were negative. This implies that all the selected banks recorded a loss during the study period except Stanbic IBTC bank. Furthermore, the returns series exhibit high kurtosis and are asymmetrically distributed since the Jacque Bera statistics p-values were all less than 5%. This finding agreed with Dikko et al. (2015) of non-normally distributed return series.

3.2. Augmented Dickey-Fuller Test for Stationarity

The statistic of the ADF shows that the series was stationary since the ADF statistic is greater than 1% critical level. Therefore, there was no need for transformation (table 2). This finding agreed with the previous studies of Dikko et al. (2015) and Ozkan P. (2004). Bruce D. (2011) and Agboola et al. (2015) used the ADF test to test the stationarity of the return series.

3.3. Autoregressive Conditional Heteroscedasticity (ARCH) Effect Test

In order to estimate the volatility, an ARCH effect test was carried out to test for the presence in the series using the Lagrange Multiplier F Statistic. The result showed the presence of ARCH effect with a p-value level of less than 1% (table 2). This finding agreed with the works of Ozkan (2004) and Bruce (2011) using Lagrange Multiplier in testing the presence of the ARCH effect, which shows the significant effect and therefore suggests the application of the volatility model.

3.4. Estimates of the Parameters of GARCH Models and Its Extension Based on Daily Stock Returns from 14 Nigerian Banks

Tables 3 and 4 presented the parameter estimates of the GARCH model and its extension estimated at three error distributions, such as Skewed normal, Skewed Student-distribution, and Skewed generalized error distribution, using daily stock returns from 14 Nigerian banks. Table 3 showed the preferred GARCH model based on skewed error distribution, indicating its log-likelihood values and AIC value. The model with the least AIC value and highest Log-likelihood value was selected as preferred for each of the banks. Table 4 showed the parameter estimate for the model so selected with respect to error distributions.

The results further showed that the returns exhibited volatility clustering. This was concluded because the GARCH term was significant in most of the models considered ($p<0.05$). This implies that small changes in the volatility of returns tend to be followed by large changes in volatility. In contrast, small changes in volatility tend to be followed by small changes in volatility.

3.5. Estimate of SV Model

The parameter estimate of SV model is presented in table 5. Monte Carlo Markov Chain (MCMC) technique was used to estimate the parameters of SV model (Kim, Shephard, and Chip, 1998). The result showed minimal and acceptable Monte Carlos Standard Error (MCSE) for the mean (μ) and returned log-variance (φ), which implies that the shocks in the log-variance are persistent in all the selected bank returns.

3.6. Forecasting Performance of the Estimated Models

The forecasting performance of the two models is evaluated and presented in table 6. The Mean Square Error (MSE) and Root Mean Square Error (RMSE) were used as recommended by Franses (2016) and Khair et al. (2017). The model with the least MSE and RMSE values was considered the most suitable for forecasting. The error indicator shows that SV minimizes the error for forecasting in a period of high or low volatility. Generally, the MSE and RMSE are smaller in the SV model than in the GARCH model {APARCH (1,1) based on Skewed Student 't' distribution} in most of the bank stock. Therefore, SV model is considered the best to predict the variability in the daily stock returns in eleven (11) banks, namely: ECO Bank, First City Monument bank, Fidelity bank, first bank, Guaranty Trust bank, Stanbic IBTC bank, Sterling bank, United Bank for Africa, Union bank, WEMA bank, and Zenith bank.

In contrast, IAPARCH (1,1) based on Skewed student 't' distribution was preferred in modeling volatility of the daily stock returns of only three (3) banks, namely: Access bank, Skye Bank, and Unity bank.

4. Conclusion

The GARCH and stochastic volatility models provide an essential tool to assist analysts while attempting to model the returns volatility. This work clearly shows that GARCH and Stochastic volatility models are highly comparable in modeling the daily returns of the selected bank stocks. However, SV model remains the preferred choice when an attempt is made to model the volatility of most of the daily returns of Nigerian bank stocks.

We found that the Skewed student 't' distribution outperformed other error distributions. APARCH (1,1) is preferred over IGARCH, TGARCH, SGARCH, and GJR-GARCH models selected in terms of fitness and forecasting power with the record of the least AIC, MSE, and RMSE value in most of the chosen bank stock returns. This finding perfectly agreed with Alberg, Shalit, and Yosef (2008).

The findings of this study correspond with that of Oscar (2016), Ezequiel and Martha (2021), Ozkan (2004), Dondukova and Liu (2021). All these studies showed and proved the superiority of SV over GARCH-type models. We further showed that SV model, though comparable with APARCH (1,1), based on skewed student t distribution, is the appropriate and best for modeling financial time series data, especially the daily stock returns of the selected Nigerian bank stocks.

5. Recommendation

Based on the findings of this study, we recommend that the stochastic volatility model should be used in modeling the returns of the chosen bank stock and that further work could be done using a multiple stochastic volatility model.

6. Acknowledgement

I acknowledge Engr. Mayen Etukudo for his love and support throughout the course of the research work. I pray to God almighty to bless him and his family.

Banks	T	Mean	Maximum	Maximum	Standard Deviation	Skewness	Kurtosis	Jacque Bera	P- Value
Access	3298	0.0000248	0.121577	0.121577	0.012374	-0.07612	9.513558	5833.285	0.000000
ETI	3298	-0.000501	0.653213	0.653213	0.024180	-8.43151	564.0747	43298512	0.000000
FCMB	3298	-0.000029	0.091698	0.091698	0.013519	-0.04194	7.178447	2400.182	0.000000

Banks	T	Mean	Maximum	Maximum	Standard Deviation	Skewness	Kurtosis	Jacque Bera	P- Value
Fidelity	3298	0.000019	0.299419	0.299419	0.015219	-0.18428	96.78682	1208731.	0.000000
First bank	3293	-0.00019	0.138303	0.138303	0.012915	0.03749	13.99675	16593.18	0.000000
Guaranty	3298	0.000071	0.138052	0.138052	0.012340	-1.00892	27.97894	86300.30	0.000000
IBTC	1942	0.000411	0.248657	0.248657	0.012953	3.68202	77.69899	455898.2	0.000000
Sky	3316	-0.00026	0.380211	0.380211	0.028190	-0.14069	96.73506	1213979.	0.000000
Sterling	3296	-0.00008	0.312214	0.312214	0.015800	0.10013	93.14301	1115944.	0.000000
UBA	3297	-0.000148	0.203675	0.203675	0.014874	-0.85000	36.59837	155472.9	0.000000
UBN	3316	-0.000193	0.727129	0.727129	0.021701	11.3332	402.1252	22081060	0.000000
UNITY	3298	-0.000179	1.000000	1.000000	0.027036	16.9689	621.1574	52667735	0.000000
WEMA	3298	-0.000204	0.390935	0.390935	0.017619	0.49598	136.5059	2449427.	0.000000
Zenith	3298	0.0000128	0.148998	0.148998	0.012063	-1.12099	30.92672	107862.2	0.000000

Table 1: Summary Statistics of Daily Stock Returns of the Selected Stocks

Banks	ADF Test Statistic	Probability Values	Comment	ARCH test F-statistic	Probability Values (F-Statistic)
Access	-48.5722	0.0001	Stationary	550.2915	0.000
ETI	-47.17395	0.0001	Stationary	829.25	0.000
FCMB	-52.64352	0.0001	Stationary	84.6764	0.000
Fidelity	-59.06472	0.0001	Stationary	828.9232	0.000
First bank	-34.25112	0.0000	Stationary	196.4195	0.000
Guaranty	-54.62437	0.0001	Stationary	244.8114	0.000
IBTC	-46.49839	0.0001	Stationary	752.76	0.000
Skye	-20.56096	0.0000	Stationary	125.4944	0.000
Sterling	-58.51137	0.0001	Stationary	858.0687	0.000
UBA	-52.84849	0.0000	Stationary	428.0211	0.000
UBN	-29.54142	0.0000	Stationary	125.68	0.000
Unity	-58.01691	0.0001	Stationary	799.25	0.000
Wema	-38.70508	0.0000	Stationary	871.407	0.000
Zenith	-37.89025	0.0000	Stationary	12.84715	0.0003

Table 2: Augmented Dickey-Fuller and the Lagrange Multiplier Tests for Stationarity and Heteroscedasticity
1% Critical = -3.44, 5% Critical = -2.86, 10% Critical = -2.57

S/N	Bank	Model	Innovation	Log Likelihood	Weighted ARCH LM Tests		AIC
					P-Value	Statistic	
1	ACCESS BANK	APARCH (1,1)	SSTD	9058.901	0.9838	0.00041	-6.8638
2	ECOBANK	EGARCH (1,1)	SGED	10,369.50	0.9773	0.00081	-6.2854
3	FCMB	EGARCH (1,1)	SGED	8,691.39	0.7829	0.07594	-7.1823
4	FIDELITY	APARCH (1,1)	SGED	8703.801	0.9844	0.00038	-6.5945
5	FIRST BANK	APARCH (1,1)	SSTD	8921.864	0.9844	0.00038	-6.7727
6	GTB	APARCH (1,1)	SGED	8300.539	0.5622	0.336	-6.5494
7	STANBIC IBTC	EGARCH (1,1)	SGED	7,608.66	0.9381	0.00604	-9.7947
8	SKYE	APARCH (1,1)	SSTD	8759.104	0.9797	0.00064	-6.9945
9	STERLING	GJRGARCH (1,1)	SSTD	8639.049	0.9844	0.00038	-6.5511
10	UBA	APARCH (1,1)	SSTD	8404.56	0.9844	0.00038	-6.3699
11	UNITY	APARCH (1,1)	SSTD	12996.22	0.9844	0.00038	-9.8545
12	UNION	APARCH (1,1)	SSTD	8313.841	0.9845	0.00038	-6.2654
13	WEMA	APARCH (1,1)	SSTD	14746.79	0.964	0.00203	-11.182
14	ZENITH	APARCH (1,1)	SSTD	8808.386	0.9844	0.00038	-6.7351

Table 3: Model Selection of GARCH (1, 1) and Its Extensions Considered Based on Simulated Returns from Skewed Distribution

Bank	Models	ID	α_0 (p-value)	α_1 (p-value)	β_1 (p-value)	γ_1 (p-value)	δ (p-value)
ACCESS	APARCH (1,1)	SSTD	0.00000 (0.96954)	0.21687 (0.00000)	0.81902 (0.00000)	0.25004 (0.00000)	0.30978 (0.00000)
ETI	EGARCH (1,1)	SGED	-0.172026 (0.00000)	0.243955 (0.00000)	0.965460 (0.00000)	0.962055 (0.00000)	
FCMB	EGARCH (1,1)	SGED	0.067742 (0.00000)	0.218451 (0.00000)	0.960368 (0.00000)	1.634385 (0.00000)	
FIDELITY	APARCH (1,1)	SSTD	0.00000 (0.992794)	0.496530 (0.00000)	0.72359 (0.00000)	0.11546 (0.00000)	0.96716 (0.00000)
First bank	APARCH (1,1)	SSTD	0.000000 (0.00000)	0.243226 (0.51633)	0.807157 (0.00000)	-0.05187 (0.24772)	0.0.33411 (0.00000)
GTB	APARCH (1,1)	SSTD	0.00867 (0.31985)	0.431883 (0.00000)	0.562379 (0.00000)	0.017485 (0.75058)	1.169177 (0.00000)
IBTC	EGARCH (1,1)	SGED	-1.140530 (0.00000)	-0.094309 (0.0000)	0.849474 (0.00000)	0.863352 (0.00000)	
Skye	APARCH (1,1)	SSTD	0.00000 (0.510903)	0.171890 (0.0000)	0.86647 (0.00000)	-0.10012 (0.052659)	0.56722 (0.00000)
STERLING	GJR GARCH (1,1)	SSTD	0.000000 (1.00000)	0.256889 (0.00000)	0.711117 (0.00000)	0.044956 (0.099296)	
UBA	APARCH (1,1)	SSTD	0.000159 (0.0000)	0.210697 (0.00000)	0.835756 (0.00000)	0.037895 (0.42695)	0.40083 (0.00000)
UBN	APARCH (1,1)	SSTD	0.000000 (0.661199)	0.285737 (0.00000)	0.755127 (0.00000)	-0.053613 (0.056467)	0.516062 (0.00000)
Unity	APARCH (1,1)	SSTD	0.000000 (1.00000)	0.21464 (0.00000)	0.55518 (0.00000)	0.42423 (0.005479)	
Wema	APARCH (1,1)	SSTD	0.000000 (0.000235)	0.221987 (0.00000)	0.808674 (0.00000)	-0.054683 (0.090456)	0.408423 (0.00000)
Zenith	APARCH (1,1)	SSTD	0.000000 (0.96693)	0.184142 (0.00000)	0.852851 (0.00000)	0.013136 (0.71927)	0.469244 (0.00000)

Table 4: Estimates of Parameter and Fitness of GARCH (1, 1) and Its Extensions Considered
Based on Daily Stock Returns from Skewed Distribution

S/N	Banks	Parameters							
		μ		φ		σ		σ^2	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	Access	-19.00	0.078	0.90	0.02	24.000	4.243	580.00	18.00
2	FCMB	-28.00	0.084	0.92	0.02	30.000	7.348	870.00	54.00
3	Fidelity	-24.00	0.078	0.99	0.02	28.000	7.211	810.00	52.00
4	ETI	-18.00	0.061	0.90	0.02	23.000	3.873	530.00	15.00
5	First bank	-18.00	0.070	0.87	0.02	22.000	3.873	490.00	15.00
6	GTBank	-9.77	0.009	0.90	0.02	0.817	0.300	0.670	0.09
7	Skye	-24.00	0.049	0.99	0.02	26.000	7.071	680.00	50.00
8	Stanbic IBTC	-41.00	0.100	0.92	0.02	32.000	8.185	1000.0	67.00
9	Sterling	-28.00	0.094	0.98	0.02	30.000	7.810	890.00	61.00
10	UBA	-18.00	0.066	0.90	0.02	24.000	4.796	570.00	23.00
11	UBN	-30.00	0.110	0.92	0.01	29.000	8.185	830.00	67.00
12	Unity	-40.00	0.140	0.91	0.01	29.000	12.247	830.00	150.00
13	Wema	-39.00	0.140	0.91	0.01	29.000	10.954	850.00	120.00
14	Zenith	-18.00	0.061	0.98	0.02	23.000	3.873	530.00	15.00

Table 5: Summary of the Estimates of the Stochastic Volatility (SV) Model for the Bank Stocks

Bank	Models	innovation	Results based on the best GARCH model family		RMSE based on Stochastic volatility		Difference (%)		Preferred Model
			MSE	RMSE	MSE	RMSE	MSE	RMSE	
ACCESS BANK	APARCH (1,1)	SSTD	1.329E-07	0.0003646	1.533E-07	0.0003916	-3.32	-6.9	APARCH (1,1)
ECOBANK	EGARCH (1,1)	SGED	4.631E-07	0.0006805	4.518E-07	0.0006722	2.49	1.24	SV
FCMB	EGARCH (1,1)	SGED	2.812E-07	0.0005303	2.515E-07	0.0005015	11.81	5.74	SV
FIDELITY	APARCH (1,1)	SSTD	3.57E-07	0.0005975	3.551E-07	0.0005959	0.55	0.27	SV
FIRST BANK	APARCH (1,1)	SSTD	4.668E-07	0.0006832	3.479E-07	0.0005899	34.16	15.83	SV
GTB	APARCH (1,1)	SSTD	4.403E-07	0.0006636	4.323E-07	0.0006575	1.85	0.92	SV
STANBIC IBTC	EGARCH (1,1)	SGED	7.445E-07	0.0008628	7.444E-07	0.0008628	0.01	0.01	SV
SKYE	APARCH (1,1)	SSTD	3.692E-07	0.0006077	3.882E-07	0.000623	-4.8	-2.47	APARCH (1,1)
STERLING	GJRGARCH (1,1)	SSTD	3.389E-07	0.0005821	3.382E-07	0.0005816	0.19	0.1	SV
UBA	APARCH (1,1)	SSTD	4.071E-07	0.0006381	4.071E-07	0.0006381	0	0	SV
UNION	APARCH (1,1)	SSTD	1.621E-07	0.0004026	1.617E-07	0.0004021	0.23	0.12	SV
UNITY	APARCH (1,1)	SSTD	9.413E-07	0.0009702	9.423E-07	0.0009707	-0.1	-0.05	APARCH (1,1)
WEMA	APARCH (1,1)	SSTD	4.789E-07	0.000692	4.789E-07	0.000692	0	0	SV
ZENITH	APARCH (1,1)	SSTD	5.419E-07	0.0007362	5.413E-07	0.0007357	0.12	0.06	SV

Table 6: Comparison of the Performance of GARCH Model Family with Stochastic Volatility Models Using MSE and RMSE

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