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Quantification and Comparison of Fractal Geometry of Complex Flow Patterns of Rock Analogue, Rock Analogue Mixed with Salt and Rock Analogue Which Flows around Marker Pens

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Abstract:

The main aim of this research is to quantify and compare the different rock analogues through complex flow patterns. The specific objectives to achieve this aim are to: reveal the sort of fractures to be developed in the complex flow; determine the effect of salt inclusion in the complex flow pattern and the flow pattern around a physical property (pens), and also deduce the significance to geology. Higher viscosity and higher density thus produced larger cracks or fractures and vice-versa. The D_f values for the fractures or cracks with salt (averagely 1.60) corresponds better to the D_f values of Takayasu, Hirata, Barton and that of the general D_f data for geophysics, than the D_f values of the mixture of starch and water (1.62) and the D_f values affected by the physical property such as pen. However, the D_f values for the fractures of the mixture of starch and water which flowed around the pens (averagely 1.61) relate better than that of the mixture of only water and starch (averagely 1.62). It may thus be inferred that any additional property to the mixture of starch and water reduces the fractal dimension.

Keywords: Fractal geometry, marker pens, rock analogue, salt, structures

1. Introduction

Fractal geometry is meant to quantify complex structures, to reveal some important information on the properties of materials and the process of forming structures and also to describe complex structures mathematically. Fractal geometry is one which is scale independent or self-similar at all scales between upper and lower fractal limits (Gillespie et al., 1993). The concept of fractals was introduced by Mandelbrot (1967) in a geological context. He noticed that the length of a rocky coastline increased as the length of the measuring rod decreased according to a power law, and thus he associated the power with a fractal dimension. The rocky coastline is observed to be scale invariant. Mandelbrot (1982) used fractal concepts to generate synthetic landscapes that looked remarkably similar to actual landscapes. The definition of a fractal distribution is that the number of objects N with a characteristic size greater than r scales with the relation $N(r) \sim r^{-D}$. $\rightarrow N = C \dots \text{equation } 1r^D$

The power determines the fractal dimension (D) of the coastline and C a constant of proportionality. The whole experiment is to compare and contrast the fractal geometry of the rate of flow of a complex rock analogue and a rock analogue diluted with salt and also rock analogue flowing around three marker pens. To achieve this objective, the following specific objectives would be achieved: to reveal the sort of fractures to be developed in the rock analogue; to determine the rate of flow of the different weight capacities of the rock analogue; the flow rate of the rock analogue with respect to the flow of magmatic rocks. Barton (1995) performed systematic studies on exposed joint and fault-trace patterns on Yucca Mountain in Nevada. The fractal statistics were good correlated and the fractal dimensions ranged from 1.4 to 1.7. Measurements in geology and geophysics that yield power law spectra have their fractal behaviour lying between $1 < D < 1.6$. Takayasu (1990) proposed the deterministic model of fracture, and ascertained the fractal dimension to be about 1.65 for the fracture pattern. The NASA Astrophysics Data System have indicated the fractal dimension D_f is about 1.5-1.6 at the central part of the Japan Arc, and decreases with distance from the centre. Hirata (1989) gave the fractal dimension of the fracture geometry of rocks as 1.6.

2. Materials and Methodology

Rock analogue is a structure that is similar in function to a rock but the chemical composition is different. The rock analogue used in this experiment was fine-grained starch ('feineSpeisestärke' from German market). The different experiments were carried out under room temperature in a Geology Laboratory at Johannes-Gutenberg Universität Mainz in Germany. At low viscosity, 40g of starch was measured, whilst 60g and 80g of starch were measured for medium and high viscosities respectively. Note that different forms of weight experiments were carried out before I could come out with this decision of 20g intervals of different forms of viscosity. The same was done for the mixture of starch and salt. On considering the amount of salt concentration to mix with the starch, I considered the different weight percentages of the

seas and that of oceans. For instance, the water of the dead sea has 29 weight % of salt (Wikipedia.org) content whilst that of the oceans (science daily.com) account for only 4%. I thus considered the weight of the salt used to mix with the starch and water in each experiment (neither as much as the dead seas concentration nor low as that of the oceans), I literally chose 10 weight percent for the salt concentration. With respect to this, low viscosity mixture was set at 40g and 4g of starch and salt respectively, 60g and 6g of starch and salt for medium viscosity respectively, the high viscosity recorded 80g and 8g of starch and salt respectively.

In performing the different forms of viscosity experiments, the low viscosity at 40g, the medium viscosity at 60g and the high viscosity at 80g of raw starch were weighed and added to the constant 100g of water. The contents were placed on the stirrer machine and allowed at different times to be homogeneously stirred. A stop clock was set immediately the starch and water contents were placed on the stirrer machine. Different times of 5, 8 and 11 minutes were recorded for low, medium and high viscosities respectively. The same stirring and timing procedures were set for the mixture of starch and salt. After the contents have been homogeneously stirred, the mixture (either raw starch or mixture of starch and salt) was poured into the plastic bottle which was being held by two metallic clamps and the contents allowed at a free space to flow along the glass plate laid on the glass box (Figure 2). The distance between the glass plate and the plastic bottle (with the contents) was measured at approximately three millimeters. These experiments were performed with the glass plate set at both the zeroing level (the plate at a totally horizontal level) and at an inclined level (approximately 5 degrees). During the inclined level flow experiments, the suspension (raw starch) was made to flow around three permanent marker pens. In this experiment, the starch contents would have to flow around the marker pens before taking the pens away.

Figure 1: Apparatuses and Substances Used for the Experiment - Measuring Glass Cup, Weighing Scale, Stirring Machine, Plastic Substance, Stop Watch and Starch

Figure 2: A Mixture of Starch and Water Flowing on a Glass Plate

2.1. Stitching the Images

The flow experiments took two days to dry up. The Nikon 70 camera was implored to take photographs of all the dried mixtures. The photos were taken at overlaps of 2.5 cm in both the column and row directions in 3 columns times 5 rows giving fifteen photos. The distance from the vertical position of the camera to the measured starch was set at 12 cm and the camera aperture of F11 at a set time of 10 seconds. These parameters were set constant for all the experiments. This is because a slight change of any of these parameters and their resolutions would affect the quality of the pattern and such result in biasing of the results. For instance, color contrast between two different photos at different positions of the same experiment or different experiments lead to inaccuracy of the images. These pictures would be set and analyzed from raw data (DSC_NEF) to tiff images (DSC_tif) either with the PTGui Software or the Adobe Photoshop Software to get all the fifteen overlapped pictures stitched together as one packed clear picture. Each of the fifteen small pictures of a particular image represents only a single fraction of the total structure.

These fifteen photos were being stitched together with the aid of both the PTGui and Adobe Photoshop software. The photographs were stitched together and converted to grey-scale images in order to separate the starch and cracks (fractures) of the rock analogue. To process the image begins with a clean-up using the Adobe Photoshop software. This is followed by the image stitching procedure using the PTGui software and this will convert the images into black and white

images. The white phase of image represents cracks or fractures whilst the black phase of the output image represents the starch. The quality of the pattern or fabrics depends on the quality of the generated resolution of the image. Overview of each experiment was photographed. I made sure that the chosen camera aperture of F11 and the set time of 10 seconds as were constant for all the photos taken for a better image resolution.

Types of Viscosity	Weight of Water (g)	Weight of Starch (g)	Weight of Salt (g)	Number of Pens	Raw Data	Tiff Image
Low or Medium or High	100	40 or 60 or 80	4 or 6 or 8	3	DSC_3996.NEF DSC_3997.NEF DSC_3998.NEF DSC_3999.NEF	DSC_3996.tif DSC_3997.tif DSC_3998.tif DSC_3999.tif

Table 1: Raw Data and Tiff Images towards Black and White Stitched Images of the Analogue of Different Weights of Starch, a Mixture of Starch and Salt as Well as Starch Flowing Around Pens

Figure 3: An Image Pattern of Fifteen Single Portions of Five Horizontal Photos and Three Vertical Photos Stitched Together

The black and white stitched phases of the image help to determine the pattern of the anisotropy of the structure using the software Mapping of Rock Forming Anisotropy (MORFA). In this mapping procedure, gliding of the window is applied to the distribution of the pattern. With MORFA analysis towards the quantification of anisotropic structures, various portions of height and width images were analyzed from the complete pattern (ranging from 1200 to 200 in my case of experiment). That means the box size of each image was defined from the largest image crop possible. The box size of the largest image crop was reduced until the results became unstable. Image for each viscosity was tested and the MORFA software made to run. The following points were applied to select good sub-image figures to be analyzed anisotropic with MORFA ANALYZE:

- The direction of the image should not be widely different from that of the others
- The correlation coefficient (R^2) of the ellipse or sphere should be high, at least 80 %
- The correlation coefficient (R^2) of the corresponding cumulative segment length points should as well be high, at least 80 %
- Good particle or point distribution on the ellipse (Wilson, 2018).

The larger the image height and width, the fewer the sub-images to be analyzed and vice-versa (Wilson, 2018). The table 2 is the calibration table for the sub-images of figure 4. The sub-images of the results were plotted using semi-log plot.

Name	Description	Height/Width (Px)	Orientation (°)	Threshold Length	Det. Coefficient R^2 (%)
60g raw starch	Medium viscosity starch mixed with water	400 * 400	134	20 – 80	50
		500 * 500	143	20 – 80	63
		600 * 600	31	20 – 80	70
		700 * 700	16	20 – 80	89
		800 * 800	16	20 – 100	89
		900 * 900	172	20 – 100	95
		1000 * 1000	12	20 – 100	92
		1200 * 1200	73	20 – 100	88

Table 2: Calibration Table of Sub-Images for MORFA Analyze

Figure 4: Semi-Log Sub-Image Plot of Low Viscosity Starch (White Phase = Cracks / Fractures)

The next step is to run the MORFA with the settings for all images and for both black (starch) and white (cracks /fractures) phases. Range of overlap for the various heights and widths of the boxes were set (Wilson, 2018). Fabrics in geo-materials and their related analogues (starch and salt) reveal patterns of complex nature. Their sizes, shapes as well as their directions also give information on the physical properties of materials. These fabrics which are mostly anisotropic in direction was quantified. To analyze and quantify the anisotropy, it must be noted that irregular areas of analysis lead to different length of scan-lines in different directions which can falsify the overall result of quantifying the anisotropy. The anisotropic directions exhibit the rate of deformation and its flow process. Anisotropy through deformation provides important information about structure forming processes and can be interpreted to be strain. Deformation processes relate to both a flow plane as well as a flow direction (Fig. 5).

Figure 5: Low Viscosity Black Phase MOR FaImage of A 40g Raw Starch at Horizontal Flow Analyzed as Starch with Its Anisotropy (Flow Direction)

To be able to really quantify anisotropy or pattern inhomogeneity, box counting is defined and applied together with window gliding method (Kruhl, 2012). This leads to quantification of fabrics or patterns of rock analogue. This method of fractal analysis was applied to the cracks and starch patterns to establish the fractal nature of the starch and crack pattern of the rock analogue. In determining the fractal geometry of this geological analogue, the box-counting method was applied.

Box-counting dimension (D_b) which is a positive real number is defined by the relation $N(r) \sim r \exp D_b$; equation (2) where D_b = box-counting dimension. $N(r)$ is the minimum number of boxes of length r necessary to cover a 2D pattern. For Euclidean objects, equation (2) defines $D_b = 1$ for a straight line and 2 for a plane, $1 < D_b < 2$ [11].

This method gathers data and uses the gathered data to analyze complex patterns. The analysis is done by breaking the set of data or image into smaller box-shaped sizes. In order to estimate the fractal dimension of the experiment, the Euclidean space containing the image pattern would have to be divided into a grid of boxes of size r . The boxes $N(r)$ which are not empty are counted (Takayasu, 1990). The procedure is repeated with the pattern image at smaller sizes (r) and each time the associated non-empty boxes are counted. Various portions of height and width images were analyzed from the complete pattern (ranging from 1200 to 200 Pixels at 100 Pixels interval, in this experiment). $N(r)$ is plotted against the

reciprocal size ($1/r$) of the squares in a double-logarithmic diagram at a scale of $1\text{cm} = 522$ pixel (Fig. 6). The required slope defines the fractal dimension (D_f).

Figure 6: Box-Count High Viscosity Horizontal Experiment of 80g Starch and 8g Salt

3. Results

During the low viscosity experiment, the rate of flow was very fast. Due to that, it did not produce a round shape of cake. This is because the resistance to flow was very low. Much water could flow out of the main cake (Fig.7a). At the medium viscosity experiment, the flow rate was not fast. It needed time to flow. It did not produce a perfect round cake but produced a better and recognizable almost round shape of cake. There was not much resistance to flow and only some water could flow out of it (Fig. 7b). For the high viscosity flow experiment, the rate of flow was very slow and as such there was a production of a round shape of cake (Fig. 7c). This was because the resistance to flow was very high. A very insignificant amount of water was formed around the main cake. It may be stated from here that: 'thick fluids have higher viscosity than thin or runny fluids' (Fossen, 2010).

The sizes of the cracks were differently observed with respect to the different forms of viscosity flow experiments (both tilted and zeroing level). It was visibly observed that the low viscosity experiments at (40g starch, mixture of 40g starch and 4g salt, and 40g starch flowing around 3 marker pens) produced smaller cracks whilst the medium viscosity (60g starch, mixture of 60g starch and 6g salt, 60g and starch flowing around 3 marker pens) and the high viscosity (80g starch, mixture of 80g starch and 8g of salt, and 80g starch flowing around 3 marker pens) experiments produced large and larger cracks respectively (figures 8, 9 and 10).

Mixing the starch and water together with a chemical substance, in this wise salt, dilutes the mixture. Salt which has low porosity, low density and is impermeable, is itself viscous and behaves like a fluid (Fossen, 2010). Figure 10 reveals darker complex image compared to other mixtures without salt. The sizes of the fractures or cracks (with respect to low, medium and high viscosities) are still small, big and bigger (as already explained above). However, the fracture sizes are small compared to the various fracture sizes of the different forms of viscosities without salt. Note that the weak and viscous nature of salt enables much wider areas to be deformed. The different weights of salt give different deformational strengths. The result is always the same as the mixture of starch and water when the mixture flows around pens since pen is a physical property (Fig. 9). That means there is no specific or clearly defined effect when the mixture flows around pens.

Figure 7: Viscosity Flow Experiments Depicting The Rate of Flow at: (A) Low; (B) Medium; and (C) High Viscosity

Figure 8: Flow Experiment of Raw Starch of: (A) Low Viscosity
(B) Medium Viscosity (C) High Viscosity

Figure 9: Flow Experiment of Raw Starch Flowing around Marker Pens at Ca. 5°
Incline of: (A) Low Viscosity (B) Medium Viscosity (C) High Viscosity

Figure 10: Flow Experiment of A Mixture of Raw Starch and
Salt Of: (A) Low Viscosity (B) Medium Viscosity (C) High Viscosity

Starch + Water	Low Viscosity	Crack	D_f	1.61
			R^2	0.996
		starch	D_f	1.91
	Medium Viscosity		R^2	0.997
		Crack	D_f	1.64
			R^2	0.996
	High Viscosity	Starch	D_f	1.88
			R^2	0.998
		Crack	D_f	1.61
Starch + Water + Salt	Low Viscosity		R^2	0.996
		Crack	D_f	1.60
			R^2	0.996
	Medium Viscosity	Starch	D_f	1.88
			R^2	0.998
		Crack	D_f	1.61
	High Viscosity		R^2	0.997
		starch	D_f	1.84
			R^2	0.998
Starch + Water + 3 Pens	Low Viscosity	Crack	D_f	1.60
			R^2	0.998
		Starch	D_f	1.89
	Medium Viscosity		R^2	0.998
		Crack	D_f	1.61
			R^2	0.997
	High Viscosity	starch	D_f	1.84
			R^2	0.998
		Crack	D_f	1.61

Table 3: Results of Box-Counting Fractal Dimensions and Correlation Values (R^2) of Raw Starch with Water, Mixture of Starch, Water and Salt, and Starch and Water Flowing around 3 Marker Pens

With reference to figure 6 and table 3, the slope of the obtained straight line gives us the fractal dimension (D_f) whilst the accuracy of these straight lines reveal the various correlation values (R^2). Different ranges of values were detected for both the slope and the accuracy of the straight lines since different phases (white for cracks and black for starch) of analyses were conducted. Also the addition of salt solutions to the starch revealed different values of slope (D_f) and accuracy (R^2). Straight lines with maximum correlation and better slope were selected. It was generally observed that the correlation coefficients, R^2 of the black phase in analyzing starch in the mixture of starch and water, ranged between 0.996 and 0.998. The corresponding fractal dimension, D_f , of this black phase analysis ranged between 1.88 and 1.91. Also, the correlation coefficients, R^2 of the black phase in analyzing starch in the mixture of starch, water and salt lied on the value 0.998. However, the black phase analysis with the mixture of starch, water and salt gave a better linear correlation. The corresponding fractal dimension of this black phase analysis ranged between 1.84 and 1.89. With the white phase analysis, the correlation coefficient of the mixture of starch and water was seen to be constant at 0.996 with fractal dimension, D_f , ranging between 1.61 and 1.64.

The D_f for the fractures or cracks of the medium viscosity (1.64) of the mixture of starch and water was found to be higher than that of both the low and high viscosities of the mixture of water and starch. However, the R^2 of the fractures of all the different classes of viscosities remained the same. Also, the D_f of the starch (background) of the medium viscosity may be seen to be lower (1.88) than that of both the low and high viscosities of the same mixture of starch and water. On the mixture of water, starch and salt, the D_f of the fractures of the medium viscosity may be seen to be higher than that of the low and the high viscosities. However, the D_f of the starch of the same mixture of medium viscosity may be seen to be lower than that of the low and high viscosities. Their R^2 of all the three classes of viscosities remained the same.

The presence of pens yielded low correlation values of 0.994 – 0.995 and 1.61 fractal dimensional value for the fractures, whilst the starch yielded 0.997 – 0.998 and 1.84 – 1.85 for both the correlation values and fractal dimensional values respectively. The relatively high box counting dimensions of 1.59 – 1.64 and 1.82 – 1.91 indicate a non-clustered distribution which argues for a homogeneous crystallization history (Peternell and Kruhl, 2009).

4. Discussions and Conclusion

The ease with which fluids flow is described in terms of their viscosity and viscous deformation implies dependence of stress on strain rate: higher stress means faster flow or more rapid strain accumulation (Fossen, 2010). In this experimental research work different forms of viscosities were determined. At low viscosity, there is higher stress and as such the flow is rapid which in turn accumulates strain. The type of flow in this wise is turbulent. However, at higher viscosity, lower stress is built up corresponding to lower strain accumulation. This results in slower movement of the starch and laminar flow is pronounced.

Considering the low, medium and high viscous fluids, the less viscous fluids lead to more flow and vice-versa. The molecular make-up plays an active role in the flow capacity. The lower the viscosity and density of the fluid, the lower the friction and resistance to flow resulting in easy flow and vice-versa. Also, the rate of viscosity and density of the fluid correspond to the size of fractures or cracks produced. Lower viscosity and lower density produce smaller cracks and vice-versa. The inclusion of salt still provided higher values for the determination coefficients or correlation values (R^2) of crack analysis (white phase) at higher image heights and widths. However, the higher R^2 values for starch analysis (black phase) were analyzed to fall at a moderate height and width of the image. Addition of salt to the water and starch mixture renders the background or image of the mixture dark. This implies that more salt reveals darker images of the complex flow, and thus the amount of salt in the starch and water mixture is directly proportional to the dark nature of the image. It may be seen that the quantity of starch in the mixture (with or without salt) defines the size of the fractures. Thus, 40g starch at low viscosity portrays smaller fractures, whilst 60g and 80g starch at medium and high viscosities respectively portray big and bigger fractures. However, inclusion of salt in the mixture increases the sizes of fractures. This means, the rate of the salt content is directly proportional to the sizes of the fractures. This is due to the fact that the viscous nature of salt enables more wide areas to be deformed.

It may be generally observed that the anisotropy of both the cracks and the starch (though diverse directions) start from the inside and sprout out in multiple directions. Almost all the images are directed to both the top and down and sometimes to the other sides. Irrespective of how bright or dark the image is, all the images reveal multi anisotropic nature. However, the darker images, due to their many sub-images analyzed, portray higher anisotropy.

It may be deduced from the box-counting results (Table 3) that the relatively high box-counting dimensions (D_f) of 1.60-1.64 (white phase, fractures) and 1.84-1.91 (black phase, background) indicate a non-clustered distribution which argues for a homogenous crystallization history (Paternell and Kruhl, 2009). The D_f of the mixture of starch and water of both the fracture/crack and starch (background) are comparatively higher than that of the mixture of starch, water and salt as well as the mixture of starch and water flowing around 3 marker pens. This is due to the fact that salt has low density, viscous and thus dilutes the mixture, whilst the mixture of starch and water flowing around 3 marker pens has also been physically diluted, hence, lower fractal dimension (D_f) values of both the starch and fractures. However, the correlation values or linear correlation (R^2) of the mixture of water, salt and starch (0.996-0.998) is higher than that of only water and starch. This reveals that the higher and better the linear correlation (R^2), the lower the value of the fractal dimension (D_f).

The D_f values for the fractures or cracks with salt (averagely 1.60) corresponds better to the D_f values of Takayasu, Hirata, Barton and that of the general D_f data for geophysics, than the D_f values of the mixture of starch and water (1.62) and the D_f values affected by the physical property such as pen. However, the D_f values for the fractures of the mixture of starch and water which flowed around the pens (averagely 1.61) relate better than that of the mixture of only water and starch (averagely 1.62). It may thus be inferred that any additional property to the mixture of starch and water reduces the fractal dimension or geometry. However, the D_f values of the mixture with salt deforms a wider range of the fractal geometry. These values on earthquakes, joints and faults give revelation of the partings of weaknesses through which the magma flows to produce magmatic rocks. During intensive earthquake the crust undergoes deformation in the form of faulting or jointing. These places of weaknesses act as channel ways for the transmission of heat from the lower layers to the upper layers of the earth crust. This influx of heat from the lower layers causes considerable melting in the upper part of the crust. The rock material which is subsequently melted forms magmatic rocks. Magmatic rocks are formed from the cooling and subsequent solidification of hot molten rock material called magma. Magma migrates along these parting planes or planes of weakness such as fault or joint to the surface of the earth.

For Euclidean objects, $D_f = 1$ for a straight line and 2 for a plane (Paternell and Kruhl, 2009). All the three different experimented rock analogues were quantified to lie between 1 and 2. This means the starch and cracks quantified values can be found between a line and a plane. The aforementioned measurements or values show a narrow range of fractal dimensions and thus do not indicate multi-fractal behavior and the structures are self-similar and also the process is a single pattern-forming process.

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