THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Hierarchical Bayesian Estimation of Random Co-efficient Dynamic Panel Data Model

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Abstract:

This research uses a Dynamic Panel Data Model (DPDM) to study how unobserved individual heterogeneity affects parameters of inference. The DPDM presumes correlation between lagged dependent variable and individual (unit) specific effects, resulting in heterogeneity among the units. The parameters of DPDM are usually estimated using the classical approach which has no control for heterogeneity of the error terms leading to non-consistent estimates of the parameters. Therefore, Hierarchical Bayesian Estimation (HBE) of unobserved individual heterogeneity of dynamic panel models was used in order to improve on a static panel model even for a panel with small, moderate, and large N. Three experiments for the individual (N) and time (T) were considered: (10, 15), (20, 20), and (100, 15). Theoretical findings are accompanied by extensive Markov Chain Monte Carlo experiments, which show that the estimator performs well and handled the complicated pattern exhibited by the data. The results show closed estimates of the parameters as the dimension of $N > T$. The performance of the HBE was assessed using Numerical Standard Error (NSE) as $N > T$ produced the least NSE.

Keywords: Dynamic panel data model, hierarchical Bayesian estimation, heterogeneous effect, Markov chain Monte Carlo experiments, numerical standard error

1. Introduction

The linear regression model with panel data is a repeated cross-section on statistical units over a given period of time. It is the pooling of observations of cross-sectional data such as households, countries, firms and so on over several time periods, Baltagi (2008). Statistics survey deals with cross-sectional data reporting each of many different units at a single point in time while time series data are used to describe a single entity, usually an economy, consumer price indices or market. Panel data have become increasingly useable and popular in both developing and developed nations, for examples, two of the popular panel datasets in the United States of America are the Panel Study of Income Dynamic (PSID) and the National Longitudinal Survey (NLS) of labour market experience.

Panel data give more informative data, less collinearity, degree of freedom, efficiency, and variability among variables. They give information on the time-ordering of cases. They are better able to discover and access effects that are simply not noticeable in pure time series or pure cross-sectional data and are more suitable for studying the dynamics of adjustment. Also, panel data allow us to construct and test more complicated behavioural models than purely cross-sectional or time-series data. Panel data models allow the researcher to understand the dynamics of adjustment. For example, Balestra and Nerlove (1966) on the dynamic demand for natural gas, Baltagi and Levin (1986) on dynamic demand for an addictive commodity like cigarettes, Holtz-Eakin et al (1988) on a dynamic wage equation, Arellano and Bond (1991) on a dynamic model of employment, Blundell et al. (1992) on a dynamic model of company investment, Islam (1995) on a dynamic model for growth convergence, and Ziliak (1997) on a dynamic lifecycle labour supply model.

A dynamic panel data model can be described as a model with a lagged dependent variable on the right-hand side of the equation of a panel data model. Dynamic panel data are exhibiting phenomenal growth and consistently becoming popular among the behavioural and social researchers. One of the advantages of dynamic panel data over cross-sectional

data is that dynamic panel data provide sufficient information about earlier time periods for dynamic relationships to be investigated rather than observation of a single point in time.

Many economic processes show dynamic adjustment over time and overlook the dynamic points of the data. This is not only a loss of relevant and vital information but can result in severe misspecification biases in the parameter estimation. A model with lagged dependent variables for many omitted variables can be controlled to a large extent.

Pesaran and Smith (1995) examined dynamic panel data models with heterogeneous autoregressive coefficients, centred on models that assume the same autoregressive coefficients for all units but allow intercepts to vary across units. They showed that not accounting for the heterogeneity produces inconsistent estimates of the mean autoregressive coefficient and pooling of observations will result in asymptotic non-consistent estimates. To address this problem, they suggested the mean group estimator which entails the estimation of the regression model unit-by-unit as a way out to this problem.

The Markov Chain Monte Carlo (MCMC) method is considered in the context of Bayesian inference to simulate a Markov Chain so as to incur posterior samples from the joint posterior distribution of parameters of interest under a certain prior probability density.

Bayesian estimation via repeated sampling from posterior distributions facilitates hierarchical modelling of dynamic panel data, whether of random effects, correlated or unstructured observed error levels or time-varying regressor effects. As noted by Davidian and Giltinan (1995), the random effects are treated as parameters in Bayesian

MCMC estimation and are ordinarily not integrated out as often done in classical approaches. In addition, one of the characteristics of the Bayesian estimation is that no conjugate priors are imposed for the individual-specific effects, that is, the effects are not necessarily required to follow a normal prior distribution to ensure that the posterior distribution falls in the same family class of the prior as with the conjugate prior assumption imposed in regular Bayesian regression model. Rather the posterior distribution form is non-analytical in nature implementing an MCMC algorithm in the Bayesian inference to estimate the model. The models with a large number of parameters to be estimated require the additional effort of the Bayesian technique called hierarchical Bayesian technique.

Hierarchical Bayesian estimation is a multi-level analysis which brings about flexibility in parameter estimation. The hierarchical Bayesian technique is premised on assuming that hierarchical prior distributions are independently drawn from the same distribution with unknown parameters. A hierarchical prior for the random coefficient (heterogeneous) model is one which assumes that parameters across the units are independent of one another and treated both the mean and the covariance as unknown parameters that require their own prior distributions, James and Stein (1960), Efron and Morris (1975) and Morris (1983).

Therefore, this study derives a hierarchical Bayesian estimator to estimate parameters of dynamic heterogeneous panel model which is an essential task in economics, suited for inference in micro panel data models

2. Methodology

2.1. Random-Coefficients Dynamic Panel Data Model

A random coefficient dynamic panel data model can be described as:

$$y_{it} = \delta_i y_{i,t-1} + \sum_{k=0}^K X_{kit} \beta_{ki} + \varepsilon_{it} \quad (2.2)$$

$$i = 1, \dots, N, \quad t = 1, \dots, T \quad \text{and } k = 0, 1, 2$$

$$y_{it} = \delta_i y_{i,t-1} + \beta_{0i} + X_{1it} \beta_{1i} + X_{2it} \beta_{2i} + \varepsilon_{it} \quad (2.3)$$

Where y is the response variable, X' is row vector of independent variables with dimension k , $i = 1, 2, \dots, N$ and the whole model contains $Nk + 1$ parameters, that is k regression coefficients for each of N individuals together with the error precision, h . β is the k -vector of regression coefficients, with the intercept, and ε is the error term, that is $\varepsilon_{it} \sim IID(0, \sigma_\varepsilon^2)$.

2.2. Likelihood Function of the Random Coefficient Dynamic Panel Data Model

The pattern of the likelihood function relies upon assumptions made about the errors. Assumptions about ε_i and X_i^* determine the pattern of the likelihood function using the definition of the Normal density as:

$$P(y | \gamma, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[-\frac{(y_i - X_i^* \gamma_i)^2}{2\sigma^2} \right] \quad (2.4)$$

Since $i \neq j$, ε_i and ε_j are independent of one another, it follows that y_i and y_j are also independent of one another. This shows that $y \sim N(X_i^* \hat{\gamma}_i, h^{-1} I_n)$, the expression for the likelihood density denoted by $p(y | \gamma, h)$, using the definition of the multivariate Normal density, we can write the likelihood function as:

$$p(y | \gamma, h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[-\frac{h}{2} (y_i - X_i^* \gamma_i)' (y_i - X_i^* \gamma_i) \right] \right\} \quad (2.5)$$

By simplification equation 2.5 could be expressed as

$$P(y | \gamma, \sigma^2) = \frac{1}{(2\pi)^{\frac{N}{2}}} \left(\{h^{\frac{k}{2}} \exp[-\frac{h}{2} (\gamma_i - \hat{\gamma}_i)' V^{-1} (\gamma_i - \hat{\gamma}_i)]\} \{h^{\frac{v}{2}} \exp[-\frac{hv}{2s^2}]\} \right) \quad (2.6)$$

The expression (2.6) after simplification suggests a normal-gamma prior for the likelihood function, Koop (2003).

2.3. Prior Distribution of the Random Coefficient Dynamic Panel Data Model

A prior is information we have about a particular study before seeing the data, we denote the independent prior by $p(\gamma_i, h)$. Therefore, from the law of independent random variables, and we have that $p(\gamma_i, h) = p(\gamma_i).p(h)$

From equation (2.6)

$p(\gamma_i) \sim \text{Normal}$ And $p(h) \sim \text{Gamma}$

$$p(\gamma_i) = \frac{1}{(2\pi)^{\frac{k}{2}}} |V_i|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\gamma_i - \underline{\gamma}_i)' V_i^{-1} (\gamma_i - \underline{\gamma}_i) \right] \quad (2.7)$$

and

$$p(h) = C_G^{-1} h^{\frac{v-2}{2}} \exp \left(\frac{-hv}{2s^2} \right) \quad (2.8)$$

2.4. Posterior Distribution of the Random Coefficient Dynamic Panel Data Model

A posterior can be described as the product of the likelihood and the prior which implies the information obtained about the parameters after seeing the data. Applying some mathematical methods, the posterior can be independent, a conjugate or not taking a common distribution form.

It is usually denoted by $p(\gamma_i, h | y_i)$.

Then, the posterior is proportional to the likelihood times the prior.

The multiplication of equation (2.5), (2.7) and (2.8) gives:

$$P(\gamma_i, h | y) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \exp \left[-\frac{h}{2} (y_i - X_i^* \gamma_i)' (y_i - X_i^* \gamma_i) \right] \cdot \frac{1}{(2\pi)^{\frac{k}{2}}} |V_i|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\gamma_i - \underline{\gamma}_i)' V_i^{-1} (\gamma_i - \underline{\gamma}_i) \right] \cdot C_G^{-1} h^{\frac{v-2}{2}} \exp \left(\frac{-hv}{2s^2} \right) \quad (2.9)$$

$$p(\gamma_i, h | y) \propto \exp \left[-\frac{1}{2} \{h(y_i - X_i^* \gamma_i)' (y_i - X_i^* \gamma_i) + (\gamma_i - \underline{\gamma}_i)' V_i^{-1} (\gamma_i - \underline{\gamma}_i)\} \right] \cdot h^{\frac{N+v-2}{2}} \exp \left(\frac{-hv}{2s^2} \right) \quad (2.10)$$

This joint posterior density for γ_i and h does not yield any well-known distributional form; so, it cannot be solved analytically but only through a posterior simulation technique.

2.5. Hierarchical Bayesian for the Random Coefficients Dynamic Panel Data Model

In the case of fully hierarchical priors, a Markov Chain Monte Carlo method (the Gibbs sampler) is employed to calculate posterior distributions. Such an approach is particularly useful in our framework since it exploits the recursive characteristics of the posterior distribution.

Assuming γ_i for $i = 1, \dots, N$ are independent draws from a normal distribution and thus,

$$\gamma_i \sim N(\mu_\gamma, V_\gamma) \quad (2.11)$$

With γ_i and γ_j being independent of one another for $i \neq j$. The hierarchical structures of the prior occur if μ_γ and V_γ are treated as unknown parameters which require their own prior information. We assumed that μ_γ and V_γ to be independent of one another with prior distribution as

$$\mu_\gamma \sim N(\underline{\mu}_\gamma, \underline{\Sigma}_\gamma) \quad (2.12)$$

and

$$V_\gamma \sim W(\underline{v}_\gamma, \underline{V}_\gamma^{-1}) \quad (2.13)$$

$\underline{\mu}_\gamma$ is now a k -vector containing the prior means for the k regression coefficients, and $\underline{V}_\gamma$ is now a $K \times K$ positive definite prior covariance matrix.

For the error precision, using the familiar Gamma prior:

$$h \sim G(\underline{s}^{-2}, \underline{v}) \quad (2.14)$$

The conditional posteriors for the γ_i 's are independent of one another, for $i = 1, \dots, N$, with

$$\gamma_i | y_i, h, \underline{\mu}_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i) \quad (2.15)$$

Where

$$\bar{V}_i = (hX_i^{*'} X_i^* + V_\gamma^{-1})' \text{ and } \bar{\gamma}_i = \bar{V}_i (hX_i^{*'} y_i + V_\gamma^{-1} \underline{\mu}_\gamma)$$

For the hierarchical parameters, $\underline{\mu}_\gamma$ and V_γ , the relevant posterior conditionals are

$$\underline{\mu}_\gamma | y, \gamma, h, V_\gamma \sim N(\bar{\underline{\mu}}_\gamma, \bar{\underline{\Sigma}}_\gamma) \text{ and } V_\gamma^{-1} | y, \gamma, h, V_\gamma, \underline{\mu}_\gamma \sim W(\bar{v}_\gamma, [\bar{v}_\gamma \bar{V}_\gamma]^{-1}) \quad (2.16)$$

Where

$$\bar{\underline{\Sigma}}_\gamma = (NV_\gamma^{-1} + \underline{\Sigma}_\gamma^{-1})^{-1}, \bar{\underline{\mu}}_\gamma = \bar{\underline{\Sigma}}_\gamma \left(V_\gamma^{-1} \sum_{i=1}^N \gamma_i + \underline{\Sigma}_\gamma^{-1} \underline{\mu}_\gamma \right), \bar{v}_\gamma = N + \underline{v}_\gamma, \bar{V}_\gamma = \sum_{i=1}^N (\gamma_i - \underline{\mu}_\gamma)(\gamma_i - \underline{\mu}_\gamma)' + \underline{V}_\gamma$$

Since $\sum_{i=1}^N \gamma_i$ is the k -vector containing the sums of the elements of γ_i .

The posterior conditional for the error precision has the form:

$$h | y, \gamma, V_\gamma, \underline{\mu}_\gamma \sim G(\bar{s}^{-2}, \bar{v}) \quad (2.17)$$

Where

$$\bar{v} = TN + \underline{v} \text{ and } \bar{s}^2 = \frac{\sum_{i=1}^N (y_i - X_i^* \gamma_i)' (y_i - X_i^* \gamma_i) + \underline{v} \underline{s}^2}{\bar{v}}.$$

Nevertheless, the posterior simulator called the Gibbs sampler, involving (2.16) and (2.17), involves only random number generation from the Normal, Gamma and Wishart distributions.

3. Analysis and Results

The analysis based on the data generated process of which the error terms are generated from normal distribution with mean zeros and variance one, that is $\varepsilon_i \sim N(0, h^{-1})$ of $\varepsilon_i \sim N(0, 1)$. We set our error precision: $h = 25$ and prior hyperparameters as $\underline{\mu}_\gamma = 0_4$, $\underline{\Sigma}_\gamma = I_4$, $\underline{V}_\gamma^{-1} = I_4$ and $\underline{v}_\gamma = 2$. Markov Chain Monte Carlo (MCMC) experiments with 10,000 replicates were made for the two scenarios.

Individual	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	-0.0198	-0.3246	2.6871	1.7403	0.0031	0.0348	0.0566	0.0838
2	-0.0197	-0.2655	2.8181	1.6056	0.0297	0.0242	0.0744	0.0509
3	-0.0297	-0.3796	2.7098	1.7918	0.0129	0.0898	0.0341	0.1353
4	-0.0272	-0.2599	2.8131	1.6440	0.0104	0.0299	0.0693	0.0125
5	-0.0219	-0.1658	2.7794	1.5998	0.0052	0.1241	0.0356	0.0567
6	-0.0046	-0.3148	2.7317	1.5776	0.0121	0.0249	0.0121	0.0789
7	-0.0381	-0.3011	2.7644	1.7434	0.0214	0.0114	0.0206	0.0869
8	-0.0007	-0.2686	2.7231	1.6182	0.0160	0.0211	0.0207	0.0384
9	-0.0175	-0.3281	2.7711	1.6381	0.0009	0.0383	0.0274	0.0184
10	-0.0120	-0.2901	2.6401	1.6063	0.0287	0.0002	0.1036	0.0502

Table 1: Posterior Mean and Posterior Standard Deviation for the First Stage Hierarchical Prior Parameters

$$\gamma_i | y_i, h, \underline{\mu}_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i) \text{ When } N < T, N=10, T=15 \quad \beta_{0i} \sim N(0, 0.25) \quad \delta_i \sim B(0, 1)$$

Individual	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.0919	-0.2448	1.8891	1.9069	0.0063	0.0384	0.0218	0.0015
2	0.1029	-0.2797	1.8367	1.9113	0.0172	0.0733	0.0742	0.0029
3	0.1106	-0.2361	1.8571	1.8325	0.025	0.0297	0.0538	0.0759
4	0.0714	-0.2231	1.9498	1.9416	0.0142	0.0168	0.0389	0.0332
5	0.0945	-0.2159	1.883	1.8708	0.0089	0.0095	0.0279	0.0376
6	0.0738	-0.2189	1.9562	1.9675	0.0119	0.0125	0.0452	0.0592
7	0.0998	-0.2461	1.8287	1.8916	0.0142	0.0398	0.0821	0.0167
8	0.075	-0.2825	1.9324	1.9431	0.0107	0.0761	0.0214	0.0347
9	0.0803	-0.1792	1.9229	1.934	0.0054	0.0271	0.0119	0.0256
10	0.8595	-0.2131	1.8923	1.9146	0.0003	0.0068	0.0185	0.0063
11	0.0878	-0.2085	1.9173	1.9183	0.0021	0.0022	0.0064	0.01
12	0.0776	-0.244	1.9586	1.9279	0.0081	0.0376	0.0476	0.0195
13	0.0685	-0.1593	1.8835	2.0242	0.0171	0.0469	0.0274	0.1159
14	0.0841	-0.1303	1.9252	1.8993	0.0016	0.076	0.0142	0.009
15	0.0986	-0.1821	1.9096	1.8143	0.0129	0.0242	0.0013	0.0941
16	0.0818	-0.1625	1.9292	1.914	0.0038	0.0438	0.0183	0.0056
17	0.0765	-0.173	1.9377	1.9519	0.0092	0.0333	0.0267	0.0435
18	0.0917	-0.1302	1.9083	1.86	0.006	0.0761	0.0026	0.0483
19	0.0891	-0.214	1.9494	1.8427	0.0035	0.0077	0.0384	0.0655
20	0.071	-0.183	1.9514	1.9004	0.0146	0.0233	0.0405	0.008

Table 2: Posterior Mean and Posterior Standard Deviation for the First Stage Hierarchical Prior Parameters

$$\gamma_i | y_i, h, \mu_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i) \text{ When } N=T, N=20, T=20, \beta_{0i} \sim N(0, 0.25) \delta_i \sim B(0, 1)$$

Ind	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
1	0.15362	0.18872	2.04768	1.88267	0.00045	0.02642	0.00073	0.00277
2	0.15097	0.14775	2.07213	1.88496	0.0022	0.01456	0.02372	0.00049
3	0.15946	0.15507	2.05688	1.84909	0.00629	0.00723	0.00847	0.03635
4	0.15875	0.21536	2.03774	1.86828	0.00558	0.05309	0.01067	0.01717
5	0.14794	0.15254	2.06859	1.90182	0.00523	0.00977	0.02018	0.01638
6	0.14678	0.22505	2.03814	1.93535	0.00639	0.06275	0.01027	0.0499
7	0.15332	0.09711	2.04999	1.88602	0.00015	0.06519	0.00159	0.00057
8	0.14923	0.16095	2.03645	1.91936	0.00394	0.00135	0.01196	0.03392
9	0.15442	0.16809	2.05282	1.88307	0.00125	0.00579	0.00441	0.00237
10	0.16241	0.13027	2.00478	1.88556	0.00925	0.03204	0.04363	0.00012
11	0.14702	0.14851	2.06981	1.89309	0.00619	0.01379	0.0214	0.00765
12	0.14896	0.15357	2.04182	1.89643	0.00421	0.00874	0.00659	0.01098
13	0.14829	0.16903	2.03806	1.93006	0.00488	0.006728	0.01035	0.04462
14	0.15394	0.17672	2.05269	1.89029	0.00077	0.01442	0.00428	0.00485
15	0.14839	0.14677	2.07135	1.90498	0.00477	0.01553	0.02294	0.01954
16	0.15733	0.14573	2.04061	1.88107	0.004165	0.01657	0.0078	0.0044
17	0.15387	0.17881	2.0243	1.91527	0.0007	0.0165	0.02411	0.02983
18	0.16199	0.20272	2.0554	1.84356	0.00882	0.04042	0.00694	0.04189
19	0.14879	0.17773	2.06405	1.89568	0.00437	0.01543	0.01564	0.01023
20	0.16316	0.20569	2.03959	1.85482	0.00992	0.04338	0.00881	0.03062
21	0.15154	0.17382	2.06851	1.87781	0.00163	0.01151	0.02009	0.0076
22	0.15346	0.15978	2.05611	1.88939	0.00029	0.00253	0.00769	0.00395
23	0.16192	0.13502	2.02164	1.8403	0.00875	0.02729	0.02677	0.04514
24	0.1557	0.18192	2.05901	1.86129	0.00253	0.01962	0.01059	0.02415
25	0.1488	0.19459	2.04605	1.90041	0.00437	0.03228	0.00236	0.01497

Ind	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
26	0.14092	0.16733	2.07606	1.92777	0.01225	0.00503	0.02765	0.04232
27	0.14733	0.18252	2.05181	1.90506	0.00584	0.02021	0.00339	0.01961
28	0.15569	0.16611	2.03188	1.89084	0.00252	0.0038	0.01653	0.00539
29	0.15072	0.16604	2.03914	1.90442	0.00245	0.00374	0.00928	0.01898
30	0.16238	0.14009	2.00993	1.85599	0.00921	0.02221	0.03848	0.02945
31	0.15053	0.16327	2.05982	1.87019	0.00264	0.00096	0.01141	0.01526
32	0.15957	0.15704	2.03877	1.87722	0.00639	0.00527	0.00964	0.00822
33	0.15799	0.16472	2.04336	1.87039	0.004829	0.00241	0.00506	0.01506
34	0.15507	0.15774	2.02475	1.87537	0.001903	0.00456	0.02366	0.01008
35	0.1584	0.12943	2.05493	1.84771	0.00523	0.03288	0.00652	0.03774
36	0.14713	0.16912	2.04802	1.90874	0.00604	0.00681	0.00039	0.02329
37	0.15539	0.16922	2.02848	1.89623	0.00223	0.00692	0.01993	0.01076
38	0.15089	0.14437	2.04723	1.88816	0.00228	0.01793	0.00118	0.00272
39	0.14783	0.18286	2.07025	1.90869	0.00534	0.02056	0.02185	0.02324
40	0.16046	0.15306	2.04109	1.85892	0.00729	0.00925	0.00731	0.02653
41	0.16303	0.14358	2.02809	1.85327	0.00986	0.01873	0.02032	0.03217
42	0.14735	0.19171	2.07113	1.89636	0.00582	0.0294	0.02272	0.01092
43	0.15035	0.15693	2.07428	1.88144	0.00282	0.00537	0.02587	0.004
44	0.14589	0.19347	2.04347	1.94193	0.00728	0.03117	0.00494	0.05649
45	0.15979	0.12356	2.04101	1.85566	0.00662	0.03871	0.0074	0.02978
46	0.15493	0.11362	2.06166	1.8628	0.00176	0.04869	0.01325	0.02264
47	0.15493	0.11075	2.04939	1.97898	0.00176	0.05156	0.00099	0.00646
48	0.15353	0.12425	2.04873	1.87744	0.00037	0.03805	0.00032	0.00801
49	0.15367	0.123	2.04204	1.88486	0.0005	0.0393	0.00637	0.00056
50	0.14613	0.15171	2.05975	1.9132	0.00704	0.01059	0.01134	0.02775
51	0.14972	0.17537	2.07028	1.87778	0.00345	0.01307	0.02187	0.00765
52	0.15508	0.16539	2.06219	1.85563	0.00191	0.00309	0.01378	0.02982
53	0.15776	0.15279	2.03541	1.85864	0.00459	0.00951	0.01299	0.02681
54	0.15146	0.116931	2.05259	1.88911	0.00171	0.04537	0.00418	0.00367
55	0.15582	0.13147	2.05182	1.87932	0.002651	0.03083	0.00341	0.00612
56	0.15128	0.20896	2.05054	1.89618	0.001891	0.04665	0.00213	0.01074
57	0.15802	0.1397	2.01205	1.89018	0.004854	0.0226	0.03636	0.00473
58	0.15015	0.15826	2.07344	1.87883	0.003024	0.00405	0.02503	0.00662
59	0.15279	0.16796	2.09096	1.87535	0.00038	0.00566	0.04255	0.01009
60	0.15753	0.15535	2.01146	1.88999	0.00436	0.00696	0.03695	0.00454
61	0.13945	0.18375	2.07304	1.93581	0.01372	0.02448	0.02463	0.05037
62	0.15524	0.16502	2.05284	1.87424	0.00204	0.00272	0.00443	0.0112
63	0.15301	0.23091	2.03968	1.89874	0.00016	0.0686	0.00873	0.01329
64	0.15427	0.16962	2.02628	1.88049	0.00109	0.00732	0.02213	0.00495
65	0.16141	0.14705	2.02606	1.86385	0.00824	0.01525	0.02235	0.02159
66	0.14596	0.17212	2.06067	1.90876	0.00728	0.00982	0.01226	0.02332
67	0.15438	0.14608	2.03629	1.88005	0.00121	0.01623	0.01212	0.00539
68	0.15383	0.19348	2.04872	1.89299	0.00067	0.03117	0.00031	0.00755

Ind	Posterior Mean				Posterior Standard Deviation			
	δ	β_0	$\beta_1(2)$	$\beta_2(3)$	δ	β_0	$\beta_1(2)$	$\beta_2(3)$
69	0.15484	0.15841	2.058	1.85708	0.00168	0.00389	0.00959	0.02837
70	0.14219	0.18054	2.03139	1.94743	0.01098	0.01824	0.01702	0.06199
71	0.14573	0.17911	2.05897	1.91842	0.00744	0.0168	0.01056	0.03298
72	0.14267	0.15767	2.08066	1.91421	0.0105	0.00464	0.03225	0.02877
73	0.15131	0.16251	2.05212	1.8823	0.00186	0.00021	0.00371	0.00314
74	0.15191	0.18081	2.07464	1.86607	0.00125	0.01851	0.02623	0.01937
75	0.15553	0.1658	2.04453	1.88396	0.00236	0.00349	0.00388	0.00149
76	0.16127	0.15848	2.04159	1.85204	0.0081	0.00383	0.00682	0.0334
77	0.15989	0.14702	2.01099	1.87598	0.00671	0.01528	0.03742	0.00947
78	0.14812	0.1624	2.06138	1.89945	0.00505	0.0001	0.01297	0.014
79	0.15451	0.17625	2.02871	1.87864	0.00134	0.01395	0.0197	0.0068
80	0.15147	0.15991	2.06091	1.88642	0.00169	0.00239	0.0125	0.00098
81	0.15767	0.16445	2.04204	1.86833	0.0045	0.00215	0.00639	0.01712
82	0.14869	0.18104	2.06886	1.88931	0.00448	0.01873	0.02045	0.00387
83	0.1575	0.16424	2.05099	1.86073	0.00433	0.00194	0.00258	0.02472
84	0.15333	0.18379	2.04865	1.89319	0.00016	0.02149	0.00024	0.00775
85	0.15334	0.12973	2.05015	1.88493	0.00017	0.03257	0.00174	0.00512
86	0.15564	0.16094	2.06112	1.86356	0.00247	0.00136	0.01271	0.02189
87	0.15172	0.14596	2.04209	1.88726	0.00145	0.01634	0.00632	0.00181
88	0.15091	0.16199	2.04249	1.90069	0.00226	0.00032	0.00592	0.01524
89	0.15731	0.13779	2.06385	1.85457	0.00414	0.02452	0.01544	0.03088
90	0.14146	0.16028	2.06584	1.94091	0.01171	0.00202	0.01743	0.05547
91	0.14903	0.13782	2.04081	1.89996	0.00414	0.02448	0.0076	0.01452
92	0.15993	0.17401	2.03856	1.86726	0.00677	0.01171	0.00986	0.01819
93	0.14868	0.16002	2.06548	1.88618	0.00449	0.00229	0.01707	0.00073
94	0.15473	0.15163	2.03696	1.88767	0.00157	0.01067	0.01145	0.00223
95	0.157	0.16749	2.04203	1.86297	0.00383	0.00519	0.00638	0.02248
96	0.14751	0.14177	2.06147	1.90708	0.00565	0.02054	0.01306	0.02164
97	0.15799	0.18319	2.01941	1.87945	0.00482	0.02089	0.029	0.00599
98	0.15861	0.18919	2.04348	1.8682	0.00544	0.02689	0.00493	0.01724
99	0.15184	0.18018	2.03704	1.89738	0.00133	0.01788	0.01138	0.01194
100	0.15746	0.14895	2.04221	1.85457	0.00429	0.01335	0.00619	0.03087

Table 3: Posterior Mean and Posterior Standard Deviation for the First Stage Hierarchical Prior Parameters

$$\gamma_i | y_i, h, \mu_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i) \text{ When } N > T, N=100, T=15, \beta_{0i} \sim N(0, 0.25) \delta_i \sim B(0, 1)$$

	N=10, T=15	N=20, T=20	N=100, T=15
$\delta \begin{cases} \text{Mean} \\ \text{NSE} \end{cases}$	-0.018960 (0.00042)	0.087397 (0.0003)	0.153492 (0.00017)
$\beta_0 \begin{cases} \text{Mean} \\ \text{NSE} \end{cases}$	-0.270528 (0.00207)	-0.213237 (0.00125)	0.163543 (0.00043)
$\beta_{1(2.0)} \begin{cases} \text{Mean} \\ \text{NSE} \end{cases}$	2.761221 (0.00171)	1.905516 (0.00108)	2.045637 (0.00055)
$\beta_{2(3.0)} \begin{cases} \text{Mean} \\ \text{NSE} \end{cases}$	1.674693 (0.00180)	1.908115 (0.00127)	2.884716 (0.00069)
V_γ	0.0033	0.0007	0.0000

Table 4: The Posterior Means and Numerical Standard Errors of the Parameter for the Second Stage of Hierarchical Prior parameters when $N < T, N=T, N > T, \beta_{0i} \sim N(0, 0.25) \delta_i \sim B(0, 1)$

Figure 1: Histogram of Posterior Mean of Parameters $\gamma_i | y, h, \mu_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i)$ For $N=10$ and $T=15$

Figure 2: Histogram of Posterior Mean of Parameters $\gamma_i | y, h, \mu_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i)$ For $N=20$ and $T=20$

Figure 3: Histogram of Posterior Mean of Parameters $\gamma_i | y, h, \mu_\gamma, V_\gamma \sim N(\bar{\gamma}_i, \bar{V}_i)$ For $N=100$ and $T=15$

4. Conclusion

This study uses a dynamic panel model to examine how unobserved individual heterogeneity affects parameters of inference. It is observed that not accounting for the heterogeneity yields non-consistent estimates of the mean of the dynamic panel data model, even with large N . Therefore, a great deal of interest was placed on hierarchical Bayesian estimation of unobserved individual heterogeneity of dynamic panel data model, in order to improve on a static panel data model of the classical approach. The theoretical findings are accompanied by extensive Markov Chain Monte Carlo (MCMC) experiments. The findings of the results with respect to the two stages of hierarchical Bayesian estimator of their performances in terms of posterior mean, posterior standard deviation, and numerical standard error which were displayed in the previous tables and figures are as follows:

- Tables 3.1-3.3 presented results for the first stage hierarchical Bayesian estimates. These results revealed the impact of each individual on the dependent variable indicating that the MCMC approach effectively handled variation among parameters across individual. The numerical standard error indicates an approximation error obtained in second stage hierarchical prior small relative to posterior standard deviation in first stage hierarchical prior error of all parameters. Also, Figures 3.1-3.3 displayed histogram graphs for the first stage hierarchical Bayesian estimates. The graph displays the individual pattern over time period which exhibit the true picture of each parameter across individuals, indicating the real influence of independent variables over dependent variable.
- The results of the second stage hierarchical Bayesian estimates presented in Table 4 revealed that as N increases the posterior mean of the slope coefficients are closer to the initial values which are set to be 2 and 3 when the data was generated. The numerical standard errors of the parameters consistently decreased as N increased for all values of T considered. The posterior estimates for δ and β_0 are obtained within the stipulated distribution range, indicating that the estimator provides good estimates for all dimension of N and T . Our finding indicated that $N > T$ option produced the least Numerical Standard Error (NSE), hence outperformed the other two experiments. The hierarchical Bayesian estimator facilitated by suitable prior information solved the problem of heterogeneity in the dynamic panel data model. Therefore, the estimator will find useful applications in panel data economic models.

5. References

- Balestra, P. and Nerlove, M. 1966. Pooling cross-section and time-series data in the estimation of dynamic model. *Econometrica* 34: 585-612
- Baltagi, B. H. 2008. *Econometric Analysis of Panel Data*, 4th Edition. John Wiley & Sons Ltd
- Baltagi, B. H. and Levin, D. 1986. Estimation dynamic demand for cigarettes using panel data: the effects of bootlegging, taxation, and advertising reconsidered. *Review of Economics and Statistics* 68: 148-155.
- Blundell, R., S. Bond, M. Devereux and F. Schiantarelli, 1992, Investment and Tobin's q : Evidence from company panel data, *Journal of Econometrics* 51, 233–257.
- Davidian, M. and Giltinan, D.M. 1995. *Nonlinear models for repeated measurement data*. New York: Chapman and Halls
- Efron, B., and Morris, C. 1975. Data analysis using Stein's estimator and its generalizations. *Journal of the American Statistical Association* 70, 311–319.
- Holtz-Eakin, D., Newey, W. and Rosen, H.S. 1988. Estimating vector autoregressions with panel data, *Econometrica* 56, 1371–1395.
- Islam, N. 1995. "Growth Empirics: A Panel Data Approach" *The Quarterly Journal of Economics*, Vol. 110(4), pp. 1127-70
- James, W., and Stein, C. 1960. Estimation with quadratic loss. In *Proceedings of Prior distributions for variance parameters in hierarchical models the Fourth Berkeley Symposium 1*, ed. J. Neyman, 361–380. Berkeley: University of California Press.
- Koop, G. 2003. *Bayesian Econometrics*. New York: John Wiley & Sons.
- Morris, C. 1983. Parametric empirical Bayes inference: theory and applications (with discussion). *Journal of the American Statistical Association* 78, 47–65.
- Pesaran, M.H. and Smith, R. 1995. "Estimation of Long-Run Relationships from Dynamic Heterogeneous Panels", *Journal of Econometrics*, 68, 79-114.
- Ziliak, J.P., 1997. Efficient estimation with panel data when instruments are predetermined: An empirical comparison of moment-condition estimators, *Journal of Business and Economic Statistics* 15, 419–431.