THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE

Application of Leslie and Lefkovitch Models to the Population Dynamics of *Blatta Orientalis*

Eze Everestus Obinwanne

Senior Lecturer, Department of Mathematics,
Michael Okpara University of Agriculture Umudike, Nigeria

Obasi Uchenna Emmanuel

Assistant Lecturer, Department of Mathematics,
Michael Okpara University of Agriculture Umudike, Nigeria

Abstract:

*In this paper, we consider the population dynamics of *Blatta Orientalis* (Oriental Cockroach) using Leslie and Lefkovitch matrix models. The two models are used to predict the nature of the population structures. Using the eigenvalues and eigenvectors approach the population structures and growth in the long run are determined.*

Keywords: Leslie model, lefkovitch model, population, dynamics, blatta orientalis

1. Introduction

The Leslie and Lefkovitch matrix models have been employed by many authors to study the population structures, demographic analysis of animal population with age, stage reproduction and survival. Horst [1], Leslie [2], Lewis [3], Usher [4] investigated population structures and how they vary with time as in Caswell [5, 6, 7], Takenori [8], Werner [9], Udaya and Eze [10], Charles worth [11], Michael and Lemonick [12], Walker [13] and the references cited therein.

Blatta Orientalis Linnaeus is a major household pest in parts of the Northwest, Midwest and Southern United States. It is sometimes referred to as the black beetle or a water bug because of its dark black appearance and tendency to harbor in damp locations. Baldwine [14]. Oriental cockroach is approximately one inch long. Male have wings covering three quarters of their body and the female has very short (rudimentary) wings. A female cockroach produces an average of eight egg capsules per life time. The egg capsule may be carried from twelve hours to five days and then deposited in a warm sheltered environment where food is readily available. The incubation period for the oriental cockroach is about forty-two to eight-one days. The female gives no assistance to the young. Nymphs go through seven molts before becoming adults which takes approximately one year. Hagenbuch et al [15]. The species of oriental cockroach tend to be seasonal, with adult appearing in spring and summer. When large number of roaches occur overcrowding can lead to the mass migration of roach species such as the German cockroach, *Blattella germanica*, the American cockroach, *Periplaneta Americana*, the Asian cockroach, *Blattella Asahinai* and the oriental cockroach. Canless [16]. Given its characteristic and nature, many authors are inspired by this insect because of its significance in creating a set of tiny robots; Michael and Lemonick [12], group-based decision making; Viegla [17], and curing of an unusual asthma; Fuller [18]. Recently, Eze and Obasi [19] employed similar models to investigate the population dynamics of a housefly which undergoes complete metamorphosis with resounding results. Furthermore, Watkinson and White [20] suggested that the two models (Leslie and Lefkovitch) can only be used for insect and not for plant dynamics.

Motivated by the above results and ongoing research in this direction, the purpose of this paper is to investigate the population dynamics of *Blatta Orientalis* using Leslie and Lefkovitch models. The investigation is based on the incomplete life cycles which have three stages: egg→nymph→ adult. Secondly, we aimed at investigating whether the result of our analysis will show similarities or otherwise with the earlier work carried out in [19]; considering the incomplete life cycle.

2. Preliminaries

Definition 2.1. Population dynamics is the study of short-term long-term changes in size and age composition of population biological environmental processes influencing those changes

Definition 2.2 Let X be a vector space such that $T: X \rightarrow X$. A scalar λ is said to be an eigenvalue of T if some vector $x \in X$, $T(x) = \lambda x$. The vector $x \in X$ is called the eigenvector corresponding to the eigenvalue λ .

Lemma 2.2. A linear combination of linear combinations is a linear combination.

Proof. Given the linear combination $c_{1,1}x_1 + \dots + c_{1,n}x_n$ through $c_{m,1}x_1 + \dots + c_{m,n}x_n$. Consider a combination of those where the d 's are scalars along with the c 's. Distributing those d 's and regrouping gives $(d_1c_{1,1} + \dots + d_m c_{m,1})x_1 + \dots + (d_1c_{1,n} + \dots + d_m c_{m,n})x_n$ which is a linear combination of the x 's.

Lemma 2.3. A composition of linear maps is linear

Proof. Let $h: v \rightarrow w$ and $g: w \rightarrow u$ then

$$\begin{aligned} goh(c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2) &= g(h(c_1 \cdot \vec{v}_1 + c_2 \cdot \vec{v}_2)) = g(c_1 \cdot h(\vec{v}_1) + c_2 \cdot h(\vec{v}_2)) \\ &= c_1 \cdot g(h(\vec{v}_1)) + c_2 \cdot g(h(\vec{v}_2)) = c_1 \cdot (goh)(\vec{v}_1) + c_2 \cdot (goh)(\vec{v}_2) \end{aligned}$$

Shows that $goh: v \rightarrow u$ preserves linear combinations.

Theorem 2.4. (Leslie theorem) The general solution of an equation such as $Nf(t) = L^t \cdot Nf(0)$ with L diagonalization is $nf_i(t) = \sum C_{ij} \lambda_j^t$ where λ_j is the eigenvalue of L and C_{ij} are constants whose values depend on the initial population vector.

Proof. See [19]

Theorem 2.5. (Lefkovich theorem) Assume for all $k_{ij} \neq 0$, there is a simultaneous non-negative symmetric continuous subfunction $k(x, y) = k(y, x) \leq k_{ij}(x, y) \leq \bar{k}$ and that Ω is connected, then $\mathfrak{D}$ is super positive.

Proof. See [21]

3. Models

3.1. Leslie Matrix Model

Leslie matrix model is a model that is based on age specific survival and fecundity rate. Assuming there are female members of a population of a housefly which is divided into n stages or classes. Let F_i = the fecundity of a female in the i^{th} class ie F_i = the average number of off springs per female in the i^{th} class. Let P_i = the probability that female in the i^{th} class will survive to become a member of the $(i + 1)^{st}$ class

$$\text{Let } x^{(k)} = \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix} = \begin{bmatrix} \text{Population of stage 1 female in year } K \\ \text{Population of stage 2 female in year } K \\ \text{Population of stage 3 female in year } K \\ \vdots \\ \text{Population of stage } n \text{ female in year } K \end{bmatrix}$$

$$\begin{aligned} \text{Then } x_1^{(k+1)} &= F_1 x_1^{(k)} + F_2 x_2^{(k)} + \dots + F_{n-1} x_{n-1}^{(k)} + F_n x_n^{(k)} \\ x_2^{(k+1)} &= P_1 x_1^{(k)} \\ x_3^{(k+1)} &= P_2 x_2^{(k)} \\ &\vdots \\ x_n^{(k+1)} &= P_{n-1} x_{n-1}^{(k)} \end{aligned}$$

Leslie matrix that describes the change in the population over time is given by

$$L = \begin{bmatrix} F_1 & F_2 & \dots & F_{n-1} & F_n \\ P_1 & 0 & \dots & 0 & 0 \\ 0 & P_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & P_{n-1} & 0 \end{bmatrix}$$

The matrix system representing the system of linear equation given in (3.1) is $x^{(k+1)} = Lx^{(k)}$

Also note that

$$(3.2) \quad x^{(k+1)} = L^{k+1} x^0$$

where x^0 is the initial population vector

For n stages, we have $x^n = L^n x^0$ where L^n denote that stages of the population that will change over time.

3.2. Lefkovich Matrix Model

Lefkovich matrix model is a model that is based on stage-specific rate. Let $\vec{n}(t)$ represent the number of individuals at stage t . The coefficient matrix which can be constructed from the probabilities of transition among the various stages is

$$A = \begin{bmatrix} P_1 & F_2 & F_3 & F_4 \\ G_1 & P_2 & 0 & 0 \\ 0 & G_2 & P_3 & 0 \\ 0 & 0 & G_3 & P_4 \end{bmatrix}$$

where P_i = the probability of surviving and staying in stage i

G_i = the probability of surviving and growing from stage i to stage i

F_i = the fertility of stage i

The matrix equation for number of individuals from one stage another is calculated as $\vec{n}(t) = A \vec{n}(t - 1)$

Also note that $\vec{n}(t) = A \vec{n}(0)$

Where $\vec{n}(0)$ is the initial population size and t is time

4. Results

We make the following assumptions using the Leslie matrix model.

- Female population of cockroach is divided into two stages each one year in length.
- Female in the first stage produces no offspring.
- Females in the first stage have 70% chance of surviving to the second stage.
- Female in the second stage produces average of three female offsprings per year but are guarantee to die after one
- Year in stage 2
- Total females in the first and second stages are about 200.

The Leslie matrix model for the above assumption is given as

$$L = \begin{bmatrix} 0 & 3 \\ 7 & 0 \end{bmatrix} \quad (4)$$

Eigenvalues of L are $\lambda_1 = -\sqrt{2.1}$ and $\lambda_2 = \sqrt{2.1}$

The corresponding eigenvectors are $e_1 = (-2.07, 1)$ and $e_2 = (2.07, 1)$

The above matrix representing population of roaches is invertible showing that the population of roaches is almost linear at the equilibrium point. This means that the population of roaches have a step by step stage in which the life cycle is defined. Since the eigen values are real and have opposite sign, we get a saddle point which is an unstable equilibrium point. The saddle point shows that the population of roaches can increase or decrease which highly depends on the nature of the population structure.

λ_2 is the dominant eigenvalue. This value determines the population growth in the long run.

λ_1 is the transient dynamics of the population. This value determines the change in the population of roaches within a short time.

e_2 is the stable age distribution. This value determines the proportion of roaches at each age within the population. Once the stable age distribution has been reached, the population undergoes exponential growth at the rate λ_2 .

e_1 is the reproduction value. This value looks how long it will take a particular roach to complete its cycle.

4.1. Determination of the Population in the Long Run

To determine the population in the long run, we express our initial population vector $x^{(0)} = (100, 100)$ as a linear combination of the eigenvectors e_1 and e_2

$$x^{(0)} = c_1 e_1 + c_2 e_2 \text{ where } c_1, c_2 \text{ are constants.} \quad (5)$$

$$\begin{aligned} (100, 100) &= c_1(-2.07, 1) + c_2(2.07, 1) \\ (100, 100) &= (-2.07c_1, c_1) + (2.07c_2, c_2) \end{aligned}$$

$$(100) = -2.07c_1 + 2.07c_2 \quad \dots (1)$$

$$(100) = c_1 + c_2 \quad \dots (2)$$

Solving equations (1) and (2) simultaneously we have

$$c_1 = 25.85 \text{ and } c_2 = 74.15$$

Equation (5) becomes

$$x^{(0)} = 25.85e_1 + 74.15e_2$$

Evaluating for $k = 0, 1, 2, \dots$ in $x^{(k+1)} = Lx^{(k)}$ we have

$$\begin{aligned} \text{For } k = 0, \quad x^{(1)} &= Lx^{(0)} \\ &= L(25.85e_1 + 74.15e_2) \\ &= 25.85Le_1 + 74.15Le_2 \\ &= (300.15, 70.035) \end{aligned} \quad (6)$$

$$\begin{aligned} \text{For } k = 1, x^{(2)} &= L^2x^{(1)} \\ &= L^2(25.85e_1 + 74.15e_2) \\ &= 25.85L^2e_1 + 74.15L^2e_2 \\ &= (210.210056, 210.25) \end{aligned} \quad (7)$$

$$\begin{aligned} \text{For } k = n, \quad x^{(n)} &= L^n x^{(0)} \\ &= L^n (25.85e_1 + 74.15e_2) \end{aligned} \quad (8)$$

Equation (8) shows that as n gets larger, both stages of the population will continue to grow due to L^n

4.2. Determination of Growth Rate for *Blatta Orientalis*

The growth rate will be determined using the three life stages of *Blatta Orientalis* which is shown in the table below.

Stage Description	Ages	Annual Survivorship	Eggs Laid at Each Stage
Egg	(6-7) weeks	0.1354	0
Nymph	2weeks-2month	0.1042	0
Adult	(9-13) month	0.9167	40

Table 1: Stage Descriptions of *Blatta Orientalis*.

Source: Field work 2016

The Lefkovitch matrix for the above table is

$$A = \begin{bmatrix} 0.1354 & 0 & 40 \\ 0.0312 & 0.1042 & 0 \\ 0 & 0.8125 & 0.9167 \end{bmatrix} \quad (9)$$

Eigenvalues for equation (9) are

$$\lambda_1 = -0.1577, \lambda_2 = -0.1577 \text{ and } \lambda_3 = 1.4716$$

The corresponding eigenvectors are

$$e_1 = \begin{bmatrix} -0.991 \\ 0.0110 \\ 0.0073 \end{bmatrix}; e_2 = \begin{bmatrix} -0.991 \\ 0.0110 \\ 0.0073 \end{bmatrix} \text{ and } e_3 = \begin{bmatrix} 0.9992 \\ 0.0228 \\ 0.0334 \end{bmatrix}$$

From the above results, λ_3 denotes the largest eigenvalue which gives the asymptotic rate of population increase. When the population has reached its asymptotic growth rate, stage structure of the population is proportional to e_3 . e_3 which is the eigenvector corresponding to the largest eigenvalue gives the stable stage structure. The components of this eigenvector give the proportion of the species in each stage in the long run. Thus, the models can be used for insects having complete and incomplete life cycle.

5. References

- i. T. J. Horst, Use of the Leslie Matrix for Assessing Environmental Impact with an Example for a Fish Population Trans, Am. Fish Soc. (1977), pp. 06: 253-257.
- ii. P. H. Leslie, On the Use of Matrices in Population Mathematics, Biometrika. (1945), pp. 33: 183-213.
- iii. E. G. Lewis, On the Generation and Growth of a Population Sankhya. (1942), pp. 6: 93-96.
- iv. M. B. Usher, A Matrix Approach to the Management of Renewable Resources with Special Reference to Selection Forest, J. Appl. Ecol, (1966), pp. 3: 335-375
- v. H. Caswell, The Dynamics of Plant Population Ecology, (2000), pp. 81: 1675-1684.
- vii. H. Caswell, Optimal Life Histories and the Maximization of Reproductive, A General Theorem for Complex Life Cycles, Ecology (1982), pp. 63: 1218- 1222.
- ix. H. Caswell, Matrix Population Models: Construction, Analysis and Interpretation, 2nd edition, Sinau Associates, Inc. Publishers, Sunderland (2000).
- x. T. Takenori and N. Hisao, An Analysis of Life History Evolution in Terms of the Density-Dependent Lefkovitch Matrix Model. (1992).
- xi. P. A. Werner and H. Caswell, Population Growth Rate and Age Versus Stage Distribution Model for Teasel (*Dispsacus Sylvestris* Huds). (1977), pp. 58:1103- 1111.
- xii. C. O Udaya, E. O. Eze and M. K. Onyema, On the Mathematical Modelling of Population Dynamics and Growth of *Clarias Glariepinus* in a Pond, IOSR Journal of Mathematics. Vol. 9, (2013), pp. 23-25.
- xiii. B. Charlesworth, Evolution in Age-Structured Population, Press Syndicate of the University of Cambridge, New York, (1980), pp. 204-224.
- xiv. D. Michael and Lemonick, Robotic Roaches do the Trick, Time Magazine (2007)
- xv. Walker, Why Cockroaches Need their Friends, BBC Nature, Retrieved 15 June 2014, (2012).
- xvi. R. W. Baldwin and T. R. Fasulo, Cockroaches. UF/IFAS CD-ROM. SW 184. (2005)
- xvii. B. E. Hagenbuch, P. G. Kochler, R. S. Patterson and R. J. Brenner, Peridomestic Cockroaches (Orthoptera: Blattidae) of Florida: their Species, Composition and Suppression. Journal of Medical Entomology. 25: (1988), pp. 377-380.
- xviii. K. M. Canless, Entomology and Nematology of Oriental Cockroach. Copyright University of Florida, (2000).
- xix. J. Viegas, Cockroaches Make Group Decision, Discovery Channel, (2006).
- xx. D. Fuller, An Unusual Asthma Cure- *Blatta Orientalis*. National Centre for Homeopathy, (2015).
- xxi. E. O. Eze and U. E. Obasi, On the Modelling of Population Dynamics of a Housefly Using Eigenvalues and Eigenvectors, Asian Journal of Applied Sciences Vol. 02, (2014), pp. 964-969.
- xxiii. Watkinson and White, Some Life History Consequences of Modular Construction in Plants, London, (1985), pp. 313: 31-51.
- xxiv. F. Lutscher and M. A. Lewis, A Mathematical Analysis of Stage-Structured Integro-difference Equation, (2003).